

Mathematica 11.3 Integration Test Results

Test results for the 413 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (2 + 3 x^2) \sqrt{5 + x^4} \, dx$$

Optimal (type 4, 208 leaves, 6 steps):

$$\frac{20}{21} x \sqrt{5 + x^4} + \frac{2}{3} x^3 \sqrt{5 + x^4} - \frac{10 x \sqrt{5 + x^4}}{\sqrt{5 + x^2}} + \frac{1}{21} x^5 (6 + 7 x^2) \sqrt{5 + x^4} +$$

$$\frac{10 \times 5^{1/4} (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5 + x^4}} - \frac{1}{21 \sqrt{5 + x^4}}$$

$$5 \times 5^{1/4} (21 + 2 \sqrt{5}) (\sqrt{5} + x^2) \sqrt{\frac{5 + x^4}{(\sqrt{5} + x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 105 leaves):

$$\frac{1}{21} \left(\frac{x (100 + 70 x^2 + 50 x^4 + 49 x^6 + 6 x^8 + 7 x^{10})}{\sqrt{5 + x^4}} + \right.$$

$$210 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$\left. 10 (-5)^{1/4} (-21 i + 2 \sqrt{5}) \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 (2 + 3 x^2) \sqrt{5 + x^4} \, dx$$

Optimal (type 4, 192 leaves, 5 steps):

$$\frac{10}{7} x \sqrt{5+x^4} + \frac{4 x \sqrt{5+x^4}}{\sqrt{5+x^2}} + \frac{1}{35} x^3 (14+15 x^2) \sqrt{5+x^4} -$$

$$\frac{4 \times 5^{1/4} (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} + \frac{1}{7 \sqrt{5+x^4}}$$

$$5^{1/4} (14 - 5 \sqrt{5}) (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 101 leaves):

$$\frac{x (250 + 70 x^2 + 125 x^4 + 14 x^6 + 15 x^8)}{35 \sqrt{5+x^4}} - 4 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$\frac{2}{7} (-5)^{1/4} (14 \text{i} + 5 \sqrt{5}) \text{EllipticF}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int (2 + 3 x^2) \sqrt{5+x^4} dx$$

Optimal (type 4, 176 leaves, 4 steps):

$$\frac{6 x \sqrt{5+x^4}}{\sqrt{5+x^2}} + \frac{1}{15} x (10+9 x^2) \sqrt{5+x^4} -$$

$$\frac{6 \times 5^{1/4} (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} + \frac{1}{3 \sqrt{5+x^4}}$$

$$5^{1/4} (9 + 2 \sqrt{5}) (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 96 leaves):

$$\frac{x (50 + 45 x^2 + 10 x^4 + 9 x^6)}{15 \sqrt{5+x^4}} - 6 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$\frac{2}{3} (-5)^{1/4} (9 \text{i} - 2 \sqrt{5}) \text{EllipticF}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2 + 3 x^2) \sqrt{5+x^4}}{x^2} dx$$

Optimal (type 4, 171 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{(2-x^2)\sqrt{5+x^4}}{x} + \frac{4x\sqrt{5+x^4}}{\sqrt{5+x^2}} - \frac{4 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} + \\
 & \frac{5^{1/4} (2+\sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}}
 \end{aligned}$$

Result (type 4, 108 leaves):

$$\begin{aligned}
 & \frac{1}{x\sqrt{5+x^4}} \left(-10 + 5x^2 - 2x^4 + x^6 - 4(-1)^{3/4} 5^{1/4} x \sqrt{5+x^4} \operatorname{EllipticE}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] - \right. \\
 & \left. 2(-5)^{1/4} (-2\operatorname{i} + \sqrt{5}) x \sqrt{5+x^4} \operatorname{EllipticF}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)
 \end{aligned}$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4} dx$$

Optimal (type 4, 192 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} + \frac{6x\sqrt{5+x^4}}{\sqrt{5+x^2}} - \\
 & \frac{6 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} + \\
 & \frac{(2+9\sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{3 \times 5^{1/4} \sqrt{5+x^4}}
 \end{aligned}$$

Result (type 4, 98 leaves):

$$\begin{aligned}
 & \frac{1}{15} \left(-\frac{5(10+45x^2+2x^4+9x^6)}{x^3\sqrt{5+x^4}} - 90(-1)^{3/4} 5^{1/4} \operatorname{EllipticE}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] + \right. \\
 & \left. 2(-5)^{1/4} (45\operatorname{i} - 2\sqrt{5}) \operatorname{EllipticF}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)
 \end{aligned}$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (2+3x^2) (5+x^4)^{3/2} dx$$

Optimal (type 4, 235 leaves, 7 steps):

$$\frac{200}{77} x \sqrt{5+x^4} + \frac{20}{13} x^3 \sqrt{5+x^4} - \frac{300 x \sqrt{5+x^4}}{13 (\sqrt{5+x^2})} + \frac{10 x^5 (78+77 x^2) \sqrt{5+x^4}}{1001} +$$

$$\frac{1}{143} x^5 (26+33 x^2) (5+x^4)^{3/2} + \frac{300 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{13 \sqrt{5+x^4}} -$$

$$\frac{1}{1001 \sqrt{5+x^4}} 50 \times 5^{1/4} (231+26 \sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 115 leaves):

$$\frac{1}{1001} \left(\frac{1}{\sqrt{5+x^4}} x (13000 + 7700 x^2 + 11050 x^4 + 11165 x^6 + 2600 x^8 + 3080 x^{10} + 182 x^{12} + 231 x^{14}) + \right.$$

$$23100 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$\left. 100 (-5)^{1/4} (-231 i + 26 \sqrt{5}) \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 (2+3 x^2) (5+x^4)^{3/2} dx$$

Optimal (type 4, 219 leaves, 6 steps):

$$\frac{300}{77} x \sqrt{5+x^4} + \frac{40 x \sqrt{5+x^4}}{3 (\sqrt{5+x^2})} + \frac{2}{231} x^3 (154+135 x^2) \sqrt{5+x^4} + \frac{1}{99} x^3 (22+27 x^2) (5+x^4)^{3/2} -$$

$$\frac{40 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{3 \sqrt{5+x^4}} + \frac{1}{231 \sqrt{5+x^4}}$$

$$10 \times 5^{1/4} (154-45 \sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 110 leaves):

$$\frac{1}{693} \left(\frac{x (13500 + 8470 x^2 + 11475 x^4 + 2464 x^6 + 2700 x^8 + 154 x^{10} + 189 x^{12})}{\sqrt{5+x^4}} - \right.$$

$$9240 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$\left. 60 (-5)^{1/4} (154 i + 45 \sqrt{5}) \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 29: Result unnecessarily involves imaginary or complex numbers.

$$\int (2 + 3 x^2) (5 + x^4)^{3/2} dx$$

Optimal (type 4, 197 leaves, 5 steps):

$$\frac{20 x \sqrt{5+x^4}}{\sqrt{5+x^2}} + \frac{2}{7} x (10+7 x^2) \sqrt{5+x^4} + \frac{1}{21} x (6+7 x^2) (5+x^4)^{3/2} -$$

$$\frac{20 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} + \frac{1}{7 \sqrt{5+x^4}}$$

$$10 \times 5^{1/4} (7+2 \sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 106 leaves):

$$\frac{x (450 + 385 x^2 + 120 x^4 + 112 x^6 + 6 x^8 + 7 x^{10})}{21 \sqrt{5+x^4}} -$$

$$20 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$\frac{20}{7} (-5)^{1/4} (7 i - 2 \sqrt{5}) \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2 + 3 x^2) (5 + x^4)^{3/2}}{x^2} dx$$

Optimal (type 4, 199 leaves, 5 steps):

$$\frac{24 x \sqrt{5+x^4}}{\sqrt{5+x^2}} + \frac{6}{35} x (25+14 x^2) \sqrt{5+x^4} - \frac{(14-3 x^2) (5+x^4)^{3/2}}{7 x} -$$

$$\frac{24 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} + \frac{1}{7 \sqrt{5+x^4}}$$

$$6 \times 5^{1/4} (14+5 \sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 125 leaves):

$$\frac{1}{35 x \sqrt{5+x^4}} \left(-1750 + 1125 x^2 - 280 x^4 + 300 x^6 + 14 x^8 + \right. \\ \left. 15 x^{10} - 840 (-1)^{3/4} 5^{1/4} x \sqrt{5+x^4} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\left(-\frac{1}{5} \right)^{1/4} x \right], -1 \right] + \right. \\ \left. 60 (-5)^{1/4} \left(14 i - 5 \sqrt{5} \right) x \sqrt{5+x^4} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\left(-\frac{1}{5} \right)^{1/4} x \right], -1 \right] \right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4} dx$$

Optimal (type 4, 201 leaves, 5 steps):

$$-\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} + \frac{36x\sqrt{5+x^4}}{\sqrt{5+x^2}} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \\ \frac{36 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{x}{5^{1/4}} \right], \frac{1}{2} \right]}{\sqrt{5+x^4}} + \frac{1}{3\sqrt{5+x^4}} \\ 2 \times 5^{1/4} (27+2\sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{x}{5^{1/4}} \right], \frac{1}{2} \right]$$

Result (type 4, 124 leaves):

$$\frac{1}{15 x^3 \sqrt{5+x^4}} \left(-250 - 1125 x^2 - 180 x^6 + 10 x^8 + 9 x^{10} - \right. \\ \left. 540 (-1)^{3/4} 5^{1/4} x^3 \sqrt{5+x^4} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\left(-\frac{1}{5} \right)^{1/4} x \right], -1 \right] + \right. \\ \left. 20 (-5)^{1/4} \left(27 i - 2 \sqrt{5} \right) x^3 \sqrt{5+x^4} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\left(-\frac{1}{5} \right)^{1/4} x \right], -1 \right] \right)$$

Problem 39: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 (2+3x^2)}{\sqrt{5+x^4}} dx$$

Optimal (type 4, 185 leaves, 5 steps):

$$\frac{2}{3} x \sqrt{5+x^4} + \frac{3}{5} x^3 \sqrt{5+x^4} - \frac{9 x \sqrt{5+x^4}}{\sqrt{5+x^2}} +$$

$$\frac{9 \times 5^{1/4} (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} - \frac{1}{6 \sqrt{5+x^4}}$$

$$5^{1/4} (27 + 2 \sqrt{5}) (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 96 leaves):

$$9 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$\frac{1}{15} \left(\frac{x (50 + 45 x^2 + 10 x^4 + 9 x^6)}{\sqrt{5+x^4}} + 5 (-5)^{1/4} (-27 \text{i} + 2 \sqrt{5}) \text{EllipticF}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (2 + 3 x^2)}{\sqrt{5+x^4}} dx$$

Optimal (type 4, 166 leaves, 4 steps):

$$x \sqrt{5+x^4} + \frac{2 x \sqrt{5+x^4}}{\sqrt{5+x^2}} - \frac{2 \times 5^{1/4} (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} +$$

$$\frac{5^{1/4} (2 - \sqrt{5}) (\sqrt{5} + x^2) \sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{2 \sqrt{5+x^4}}$$

Result (type 4, 71 leaves):

$$x \sqrt{5+x^4} - 2 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] +$$

$$(-5)^{1/4} (2 \text{i} + \sqrt{5}) \text{EllipticF}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3 x^2}{\sqrt{5+x^4}} dx$$

Optimal (type 4, 155 leaves, 3 steps):

$$\frac{3 x \sqrt{5+x^4}}{\sqrt{5+x^2}} - \frac{3 \times 5^{1/4} (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}} +$$

$$\frac{(2+3\sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{2 \times 5^{1/4} \sqrt{5+x^4}}$$

Result (type 4, 62 leaves):

$$\left(-\frac{1}{5}\right)^{1/4} \left(-3 i \sqrt{5} \text{EllipticE}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] + \right.$$

$$\left. (-2+3 i \sqrt{5}) \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]\right)$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x^2}{x^2 \sqrt{5+x^4}} dx$$

Optimal (type 4, 173 leaves, 4 steps):

$$-\frac{2\sqrt{5+x^4}}{5x} + \frac{2x\sqrt{5+x^4}}{5(\sqrt{5+x^2})} - \frac{2(\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{5^{3/4} \sqrt{5+x^4}} +$$

$$\frac{(2+3\sqrt{5}) (\sqrt{5+x^2}) \sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{2 \times 5^{3/4} \sqrt{5+x^4}}$$

Result (type 4, 81 leaves):

$$\frac{1}{5} \left(-\frac{2\sqrt{5+x^4}}{x} - 2(-1)^{3/4} 5^{1/4} \text{EllipticE}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] - \right.$$

$$\left. (-5)^{1/4} (-2 i + 3 \sqrt{5}) \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]\right)$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x^2}{x^4 \sqrt{5+x^4}} dx$$

Optimal (type 4, 189 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{5+x^4}}{15x^3} - \frac{3\sqrt{5+x^4}}{5x} + \frac{3x\sqrt{5+x^4}}{5(\sqrt{5+x^2})} - \frac{3(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{5^{3/4}\sqrt{5+x^4}} \\
 & \frac{(2-9\sqrt{5})(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{30 \times 5^{1/4} \sqrt{5+x^4}}
 \end{aligned}$$

Result (type 4, 97 leaves):

$$\begin{aligned}
 & \frac{1}{75} \left(-\frac{5(10+45x^2+2x^4+9x^6)}{x^3\sqrt{5+x^4}} - 45(-1)^{3/4}5^{1/4} \operatorname{EllipticE}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right], -1\right] + \right. \\
 & \left. (-5)^{1/4}(45\operatorname{i}+2\sqrt{5}) \operatorname{EllipticF}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right], -1\right] \right)
 \end{aligned}$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}} dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}} - \frac{1}{5}x\sqrt{5+x^4} + \frac{9x\sqrt{5+x^4}}{2(\sqrt{5+x^2})} - \\
 & \frac{9 \times 5^{1/4}(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{2\sqrt{5+x^4}} + \\
 & \frac{(2+9\sqrt{5})(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{4 \times 5^{1/4} \sqrt{5+x^4}}
 \end{aligned}$$

Result (type 4, 85 leaves):

$$\begin{aligned}
 & \frac{1}{10} \left(-\frac{5x(2+3x^2)}{\sqrt{5+x^4}} - 45(-1)^{3/4}5^{1/4} \operatorname{EllipticE}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right], -1\right] + \right. \\
 & \left. (-5)^{1/4}(45\operatorname{i}-2\sqrt{5}) \operatorname{EllipticF}\left[\operatorname{i} \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right], -1\right] \right)
 \end{aligned}$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}} dx$$

Optimal (type 4, 177 leaves, 4 steps):

$$-\frac{x(15-2x^2)}{10\sqrt{5+x^4}} - \frac{x\sqrt{5+x^4}}{5(\sqrt{5+x^2})} + \frac{(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{5^{3/4}\sqrt{5+x^4}} -$$

$$\frac{(2-3\sqrt{5})(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{4 \times 5^{3/4}\sqrt{5+x^4}}$$

Result (type 4, 85 leaves):

$$\frac{1}{10} \left(\frac{x(-15+2x^2)}{\sqrt{5+x^4}} + 2(-1)^{3/4} 5^{1/4} \text{EllipticE}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] - \right.$$

$$\left. (-5)^{1/4} (2\text{i} + 3\sqrt{5}) \text{EllipticF}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}} dx$$

Optimal (type 4, 180 leaves, 4 steps):

$$\frac{x(2+3x^2)}{10\sqrt{5+x^4}} - \frac{3x\sqrt{5+x^4}}{10(\sqrt{5+x^2})} + \frac{3(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{2 \times 5^{3/4}\sqrt{5+x^4}} +$$

$$\frac{(2-3\sqrt{5})(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{20 \times 5^{1/4}\sqrt{5+x^4}}$$

Result (type 4, 86 leaves):

$$\frac{1}{50} \left(\frac{5x(2+3x^2)}{\sqrt{5+x^4}} + 15(-1)^{3/4} 5^{1/4} \text{EllipticE}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] - \right.$$

$$\left. (-5)^{1/4} (15\text{i} + 2\sqrt{5}) \text{EllipticF}\left[\text{i ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}} dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$\frac{\frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} + \frac{3x\sqrt{5+x^4}}{25(\sqrt{5+x^2})} - \frac{3(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{5 \times 5^{3/4} \sqrt{5+x^4}} + \frac{3(2+\sqrt{5})(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{20 \times 5^{3/4} \sqrt{5+x^4}}$$

Result (type 4, 108 leaves):

$$-\frac{1}{50x\sqrt{5+x^4}} \left(20 - 15x^2 + 6x^4 + 6(-1)^{3/4} 5^{1/4} x \sqrt{5+x^4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] + 3(-5)^{1/4} (-2i + \sqrt{5}) x \sqrt{5+x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x^2}{x^4(5+x^4)^{3/2}} dx$$

Optimal (type 4, 214 leaves, 6 steps):

$$\frac{\frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} + \frac{9x\sqrt{5+x^4}}{50(\sqrt{5+x^2})} - \frac{9(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{10 \times 5^{3/4} \sqrt{5+x^4}} + \frac{(27-2\sqrt{5})(\sqrt{5+x^2})\sqrt{\frac{5+x^4}{(\sqrt{5+x^2})^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{60 \times 5^{3/4} \sqrt{5+x^4}}$$

Result (type 4, 119 leaves):

$$-\frac{1}{150x^3\sqrt{5+x^4}} \left(20 + 90x^2 + 10x^4 + 27x^6 + 27(-1)^{3/4} 5^{1/4} x^3 \sqrt{5+x^4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] - (-5)^{1/4} (27i + 2\sqrt{5}) x^3 \sqrt{5+x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right] \right)$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int x^5 (d+e x^2) (1+2x^2+x^4)^5 dx$$

Optimal (type 1, 63 leaves, 4 steps):

$$\frac{1}{22} (d-e) (1+x^2)^{11} - \frac{1}{24} (2d-3e) (1+x^2)^{12} + \frac{1}{26} (d-3e) (1+x^2)^{13} + \frac{1}{28} e (1+x^2)^{14}$$

Result (type 1, 153 leaves):

$$\frac{d x^6}{6} + \frac{1}{8} (10d+e) x^8 + \frac{1}{2} (9d+2e) x^{10} + \frac{5}{4} (8d+3e) x^{12} + \frac{15}{7} (7d+4e) x^{14} + \frac{21}{8} (6d+5e) x^{16} + \frac{7}{3} (5d+6e) x^{18} + \frac{3}{2} (4d+7e) x^{20} + \frac{15}{22} (3d+8e) x^{22} + \frac{5}{24} (2d+9e) x^{24} + \frac{1}{26} (d+10e) x^{26} + \frac{e x^{28}}{28}$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int x^3 (d+e x^2) (1+2 x^2+x^4)^5 dx$$

Optimal (type 1, 45 leaves, 4 steps):

$$-\frac{1}{22} (d-e) (1+x^2)^{11} + \frac{1}{24} (d-2e) (1+x^2)^{12} + \frac{1}{26} e (1+x^2)^{13}$$

Result (type 1, 151 leaves):

$$\frac{d x^4}{4} + \frac{1}{6} (10d+e) x^6 + \frac{5}{8} (9d+2e) x^8 + \frac{3}{2} (8d+3e) x^{10} + \frac{5}{2} (7d+4e) x^{12} + 3 (6d+5e) x^{14} + \frac{21}{8} (5d+6e) x^{16} + \frac{5}{3} (4d+7e) x^{18} + \frac{3}{4} (3d+8e) x^{20} + \frac{5}{22} (2d+9e) x^{22} + \frac{1}{24} (d+10e) x^{24} + \frac{e x^{26}}{26}$$

Problem 60: Result more than twice size of optimal antiderivative.

$$\int x (d+e x^2) (1+2 x^2+x^4)^5 dx$$

Optimal (type 1, 29 leaves, 4 steps):

$$\frac{1}{22} (d-e) (1+x^2)^{11} + \frac{1}{24} e (1+x^2)^{12}$$

Result (type 1, 149 leaves):

$$\frac{d x^2}{2} + \frac{1}{4} (10d+e) x^4 + \frac{5}{6} (9d+2e) x^6 + \frac{15}{8} (8d+3e) x^8 + 3 (7d+4e) x^{10} + \frac{7}{2} (6d+5e) x^{12} + 3 (5d+6e) x^{14} + \frac{15}{8} (4d+7e) x^{16} + \frac{5}{6} (3d+8e) x^{18} + \frac{1}{4} (2d+9e) x^{20} + \frac{1}{22} (d+10e) x^{22} + \frac{e x^{24}}{24}$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int x^5 (1+x^2) (1+2 x^2+x^4)^5 dx$$

Optimal (type 1, 34 leaves, 4 steps):

$$\frac{1}{24} (1+x^2)^{12} - \frac{1}{13} (1+x^2)^{13} + \frac{1}{28} (1+x^2)^{14}$$

Result (type 1, 85 leaves):

$$\frac{x^6}{6} + \frac{11 x^8}{8} + \frac{11 x^{10}}{2} + \frac{55 x^{12}}{4} + \frac{165 x^{14}}{7} + \frac{231 x^{16}}{8} + \frac{77 x^{18}}{3} + \frac{33 x^{20}}{2} + \frac{15 x^{22}}{2} + \frac{55 x^{24}}{24} + \frac{11 x^{26}}{26} + \frac{x^{28}}{28}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int x^3 (1+x^2) (1+2x^2+x^4)^5 dx$$

Optimal (type 1, 23 leaves, 4 steps):

$$-\frac{1}{24} (1+x^2)^{12} + \frac{1}{26} (1+x^2)^{13}$$

Result (type 1, 83 leaves):

$$\frac{x^4}{4} + \frac{11 x^6}{6} + \frac{55 x^8}{8} + \frac{33 x^{10}}{2} + \frac{55 x^{12}}{2} + 33 x^{14} + \frac{231 x^{16}}{8} + \frac{55 x^{18}}{3} + \frac{33 x^{20}}{4} + \frac{5 x^{22}}{2} + \frac{11 x^{24}}{24} + \frac{x^{26}}{26}$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (2+3x^2) \sqrt{3+5x^2+x^4} dx$$

Optimal (type 4, 322 leaves, 6 steps):

$$\begin{aligned} & -\frac{1924 x (5 + \sqrt{13} + 2 x^2)}{105 \sqrt{3 + 5 x^2 + x^4}} + \frac{13}{3} x \sqrt{3 + 5 x^2 + x^4} - \frac{26}{35} x^3 \sqrt{3 + 5 x^2 + x^4} + \\ & \frac{1}{21} x^5 (11 + 7 x^2) \sqrt{3 + 5 x^2 + x^4} + \left(962 \sqrt{\frac{2}{3} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} \right. \\ & \left. (6 + (5 + \sqrt{13}) x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \frac{1}{6} (-13 + 5 \sqrt{13})\right] \right) / \\ & \left(105 \sqrt{3 + 5 x^2 + x^4} \right) - \left(13 \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \frac{1}{6} (-13 + 5 \sqrt{13})\right] \right) / \left(\sqrt{6 (5 + \sqrt{13})} \sqrt{3 + 5 x^2 + x^4} \right) \end{aligned}$$

Result (type 4, 237 leaves):

$$\left(2730 x + 4082 x^3 + 460 x^5 + 604 x^7 + 460 x^9 + 70 x^{11} - 1924 i \sqrt{2} (-5 + \sqrt{13}) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right. \\ \left. \sqrt{5 + \sqrt{13} + 2 x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] + \right. \\ \left. 13 i \sqrt{2} (-635 + 148 \sqrt{13}) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\ \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right) / \left(210 \sqrt{3 + 5 x^2 + x^4} \right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 (2 + 3 x^2) \sqrt{3 + 5 x^2 + x^4} dx$$

Optimal (type 4, 305 leaves, 5 steps):

$$\frac{1247 x (5 + \sqrt{13} + 2 x^2)}{210 \sqrt{3 + 5 x^2 + x^4}} - \frac{4}{3} x \sqrt{3 + 5 x^2 + x^4} + \frac{1}{35} x^3 (29 + 15 x^2) \sqrt{3 + 5 x^2 + x^4} - \\ \left(1247 \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \right. \\ \left. \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right] \right) / \left(210 \sqrt{3 + 5 x^2 + x^4} \right) + \\ \frac{1}{\sqrt{3 + 5 x^2 + x^4}} 2 \sqrt{\frac{2}{3 (5 + \sqrt{13})}} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \\ \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right]$$

Result (type 4, 234 leaves):

$$\left(4 x \left(-420 - 439 x^2 + 430 x^4 + 312 x^6 + 45 x^8 \right) + 1247 i \sqrt{2} \left(-5 + \sqrt{13} \right) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right. \\ \left. \sqrt{5 + \sqrt{13} + 2 x^2} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] - \right. \\ \left. i \sqrt{2} \left(-5395 + 1247 \sqrt{13} \right) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\ \left. \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right) / \left(420 \sqrt{3 + 5 x^2 + x^4} \right)$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int (2 + 3 x^2) \sqrt{3 + 5 x^2 + x^4} dx$$

Optimal (type 4, 279 leaves, 4 steps):

$$-\frac{23 x \left(5 + \sqrt{13} + 2 x^2 \right)}{15 \sqrt{3 + 5 x^2 + x^4}} + \frac{1}{15} x \left(25 + 9 x^2 \right) \sqrt{3 + 5 x^2 + x^4} + \\ \frac{1}{15 \sqrt{3 + 5 x^2 + x^4}} 23 \sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} \sqrt{\frac{6 + \left(5 - \sqrt{13} \right) x^2}{6 + \left(5 + \sqrt{13} \right) x^2}} \\ \left(6 + \left(5 + \sqrt{13} \right) x^2 \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} x \right], \frac{1}{6} \left(-13 + 5 \sqrt{13} \right) \right] + \\ \left(\sqrt{\frac{6 + \left(5 - \sqrt{13} \right) x^2}{6 + \left(5 + \sqrt{13} \right) x^2}} \left(6 + \left(5 + \sqrt{13} \right) x^2 \right) \right. \\ \left. \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} x \right], \frac{1}{6} \left(-13 + 5 \sqrt{13} \right) \right] \right) / \left(\sqrt{6 \left(5 + \sqrt{13} \right)} \sqrt{3 + 5 x^2 + x^4} \right)$$

Result (type 4, 229 leaves):

$$\frac{1}{30 \sqrt{3+5 x^2+x^4}} \left(2 x \left(75+152 x^2+70 x^4+9 x^6 \right)-23 i \sqrt{2} \left(-5+\sqrt{13} \right) \sqrt{\frac{-5+\sqrt{13}-2 x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2 x^2} \right. \\ \left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{2}{5+\sqrt{13}}} x \right], \frac{19}{6}+\frac{5 \sqrt{13}}{6} \right]+i \sqrt{2} \left(-130+23 \sqrt{13} \right) \right. \\ \left. \sqrt{\frac{-5+\sqrt{13}-2 x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2 x^2} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{2}{5+\sqrt{13}}} x \right], \frac{19}{6}+\frac{5 \sqrt{13}}{6} \right] \right)$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(2+3 x^2 \right) \sqrt{3+5 x^2+x^4}}{x^2} dx$$

Optimal (type 4, 284 leaves, 4 steps):

$$\frac{9 x \left(5+\sqrt{13}+2 x^2 \right)}{2 \sqrt{3+5 x^2+x^4}}-\frac{\left(2-x^2 \right) \sqrt{3+5 x^2+x^4}}{x}- \\ \frac{1}{2 \sqrt{3+5 x^2+x^4}} 3 \sqrt{\frac{3}{2} \left(5+\sqrt{13} \right)} \sqrt{\frac{6+\left(5-\sqrt{13} \right) x^2}{6+\left(5+\sqrt{13} \right) x^2}} \left(6+\left(5+\sqrt{13} \right) x^2 \right) \\ \text{EllipticE} \left[\text{ArcTan} \left[\sqrt{\frac{1}{6} \left(5+\sqrt{13} \right)} x \right], \frac{1}{6} \left(-13+5 \sqrt{13} \right) \right]+ \\ \frac{1}{\sqrt{3+5 x^2+x^4}} 8 \sqrt{\frac{2}{3 \left(5+\sqrt{13} \right)}} \sqrt{\frac{6+\left(5-\sqrt{13} \right) x^2}{6+\left(5+\sqrt{13} \right) x^2}} \left(6+\left(5+\sqrt{13} \right) x^2 \right) \\ \text{EllipticF} \left[\text{ArcTan} \left[\sqrt{\frac{1}{6} \left(5+\sqrt{13} \right)} x \right], \frac{1}{6} \left(-13+5 \sqrt{13} \right) \right]$$

Result (type 4, 231 leaves):

$$\frac{1}{4 x \sqrt{3+5 x^2+x^4}}$$

$$\left(4 (-6-7 x^2+3 x^4+x^6) + 9 i \sqrt{2} (-5+\sqrt{13}) x \sqrt{\frac{-5+\sqrt{13}-2 x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2 x^2} \right.$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}} x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right] - i \sqrt{2} (-13+9\sqrt{13}) x$$

$$\left. \sqrt{\frac{-5+\sqrt{13}-2 x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}} x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right] \right)$$

Problem 155: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2+3 x^2) \sqrt{3+5 x^2+x^4}}{x^4} dx$$

Optimal (type 4, 305 leaves, 5 steps):

$$\frac{32 x (5+\sqrt{13}+2 x^2)}{9 \sqrt{3+5 x^2+x^4}} - \frac{64 \sqrt{3+5 x^2+x^4}}{9 x} - \frac{(2-9 x^2) \sqrt{3+5 x^2+x^4}}{3 x^3} -$$

$$\frac{1}{9 \sqrt{3+5 x^2+x^4}} 16 \sqrt{\frac{2}{3} (5+\sqrt{13})} \sqrt{\frac{6+(5-\sqrt{13}) x^2}{6+(5+\sqrt{13}) x^2}}$$

$$(6+(5+\sqrt{13}) x^2) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5+\sqrt{13})} x\right], \frac{1}{6} (-13+5\sqrt{13})\right] +$$

$$\left(49 \sqrt{\frac{6+(5-\sqrt{13}) x^2}{6+(5+\sqrt{13}) x^2}} (6+(5+\sqrt{13}) x^2) \right.$$

$$\left. \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5+\sqrt{13})} x\right], \frac{1}{6} (-13+5\sqrt{13})\right] \right) / \left(3 \sqrt{6(5+\sqrt{13})} \sqrt{3+5 x^2+x^4} \right)$$

Result (type 4, 237 leaves):

$$\left(-2 (18 + 141 x^2 + 191 x^4 + 37 x^6) + 32 i \sqrt{2} (-5 + \sqrt{13}) x^3 \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right. \\ \left. \sqrt{5 + \sqrt{13} + 2 x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] - \right. \\ \left. i \sqrt{2} (-13 + 32 \sqrt{13}) x^3 \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\ \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right) / \left(18 x^3 \sqrt{3 + 5 x^2 + x^4} \right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (2 + 3 x^2) (3 + 5 x^2 + x^4)^{3/2} dx$$

Optimal (type 4, 356 leaves, 7 steps):

$$\frac{176723 x (5 + \sqrt{13} + 2 x^2)}{4290 \sqrt{3 + 5 x^2 + x^4}} - \frac{4210}{429} x \sqrt{3 + 5 x^2 + x^4} + \frac{1251}{715} x^3 \sqrt{3 + 5 x^2 + x^4} - \\ \frac{1}{429} x^5 (283 + 272 x^2) \sqrt{3 + 5 x^2 + x^4} + \frac{1}{143} x^5 (71 + 33 x^2) (3 + 5 x^2 + x^4)^{3/2} - \\ \left(176723 \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \right. \\ \left. \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right] \right) / \left(4290 \sqrt{3 + 5 x^2 + x^4} \right) + \\ \left(2105 \sqrt{\frac{2}{3 (5 + \sqrt{13})}} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right] \right) / \left(143 \sqrt{3 + 5 x^2 + x^4} \right)$$

Result (type 4, 249 leaves):

$$\frac{1}{8580 \sqrt{3+5 x^2+x^4}} \left(4 x \left(-63150 - 93991 x^2 + 3055 x^4 + 29003 x^6 + 39650 x^8 + 24635 x^{10} + 6015 x^{12} + 495 x^{14} \right) + 176723 i \sqrt{2} \left(-5 + \sqrt{13} \right) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\ \left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] - i \sqrt{2} \left(-757315 + 176723 \sqrt{13} \right) \right. \\ \left. \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right)$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 (2 + 3 x^2) (3 + 5 x^2 + x^4)^{3/2} dx$$

Optimal (type 4, 331 leaves, 6 steps):

$$-\frac{49949 x (5 + \sqrt{13} + 2 x^2)}{3465 \sqrt{3 + 5 x^2 + x^4}} + \frac{353}{99} x \sqrt{3 + 5 x^2 + x^4} - \\ \frac{x^3 (911 + 890 x^2) \sqrt{3 + 5 x^2 + x^4}}{1155} + \frac{1}{99} x^3 (67 + 27 x^2) (3 + 5 x^2 + x^4)^{3/2} + \\ \left(49949 \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \right. \\ \left. \text{EllipticE} \left[\text{ArcTan} \left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right] \right) / (3465 \sqrt{3 + 5 x^2 + x^4}) - \\ \left(353 \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \text{EllipticF} \left[\text{ArcTan} \left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \right. \right. \\ \left. \left. \frac{1}{6} (-13 + 5 \sqrt{13}) \right] \right) / \left(33 \sqrt{6 (5 + \sqrt{13})} \sqrt{3 + 5 x^2 + x^4} \right)$$

Result (type 4, 244 leaves):

$$\frac{1}{6930 \sqrt{3+5 x^2+x^4}} \left(2 x \left(37065 + 74681 x^2 + 69535 x^4 + 84962 x^6 + 50075 x^8 + 11795 x^{10} + 945 x^{12} \right) - \right.$$

$$49949 i \sqrt{2} \left(-5 + \sqrt{13} \right) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2}$$

$$\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] + i \sqrt{2} \left(-212680 + 49949 \sqrt{13} \right)$$

$$\left. \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right)$$

Problem 165: Result unnecessarily involves imaginary or complex numbers.

$$\int (2 + 3 x^2) (3 + 5 x^2 + x^4)^{3/2} dx$$

Optimal (type 4, 308 leaves, 5 steps):

$$\frac{203 x \left(5 + \sqrt{13} + 2 x^2 \right)}{30 \sqrt{3 + 5 x^2 + x^4}} - \frac{1}{15} x \left(5 + 12 x^2 \right) \sqrt{3 + 5 x^2 + x^4} +$$

$$\frac{1}{3} x \left(3 + x^2 \right) \left(3 + 5 x^2 + x^4 \right)^{3/2} - \frac{1}{30 \sqrt{3 + 5 x^2 + x^4}} 203 \sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} \sqrt{\frac{6 + \left(5 - \sqrt{13} \right) x^2}{6 + \left(5 + \sqrt{13} \right) x^2}}$$

$$\left(6 + \left(5 + \sqrt{13} \right) x^2 \right) \text{EllipticE} \left[\text{ArcTan} \left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} x \right], \frac{1}{6} \left(-13 + 5 \sqrt{13} \right) \right] +$$

$$\frac{1}{\sqrt{3 + 5 x^2 + x^4}} 5 \sqrt{\frac{2}{3 \left(5 + \sqrt{13} \right)}} \sqrt{\frac{6 + \left(5 - \sqrt{13} \right) x^2}{6 + \left(5 + \sqrt{13} \right) x^2}} \left(6 + \left(5 + \sqrt{13} \right) x^2 \right)$$

$$\text{EllipticF} \left[\text{ArcTan} \left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} x \right], \frac{1}{6} \left(-13 + 5 \sqrt{13} \right) \right]$$

Result (type 4, 239 leaves):

$$\frac{1}{60 \sqrt{3+5 x^2+x^4}}$$

$$\left(4 x \left(120 + 434 x^2 + 550 x^4 + 293 x^6 + 65 x^8 + 5 x^{10} \right) + 203 i \sqrt{2} \left(-5 + \sqrt{13} \right) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right.$$

$$\left. \sqrt{5 + \sqrt{13} + 2 x^2} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] - i \sqrt{2} \left(-715 + 203 \sqrt{13} \right) \right.$$

$$\left. \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right)$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2} dx$$

Optimal (type 4, 312 leaves, 5 steps):

$$\frac{412 x \left(5 + \sqrt{13} + 2 x^2 \right)}{35 \sqrt{3+5 x^2+x^4}} + \frac{1}{35} x \left(655 + 129 x^2 \right) \sqrt{3+5 x^2+x^4} -$$

$$\frac{\left(14 - 3 x^2 \right) \left(3 + 5 x^2 + x^4 \right)^{3/2}}{7 x} - \frac{1}{35 \sqrt{3+5 x^2+x^4}} 206 \sqrt{\frac{2}{3} \left(5 + \sqrt{13} \right)} \sqrt{\frac{6 + \left(5 - \sqrt{13} \right) x^2}{6 + \left(5 + \sqrt{13} \right) x^2}}$$

$$\left(6 + \left(5 + \sqrt{13} \right) x^2 \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} x \right], \frac{1}{6} \left(-13 + 5 \sqrt{13} \right) \right] +$$

$$\frac{1}{\sqrt{3+5 x^2+x^4}} 19 \sqrt{\frac{3}{2 \left(5 + \sqrt{13} \right)}} \sqrt{\frac{6 + \left(5 - \sqrt{13} \right) x^2}{6 + \left(5 + \sqrt{13} \right) x^2}} \left(6 + \left(5 + \sqrt{13} \right) x^2 \right)$$

$$\operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13} \right)} x \right], \frac{1}{6} \left(-13 + 5 \sqrt{13} \right) \right]$$

Result (type 4, 235 leaves):

$$\left(-1260 + 3884 x^4 + 2130 x^6 + 418 x^8 + 30 x^{10} + 412 i \sqrt{2} (-5 + \sqrt{13}) x \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right. \\ \left. \sqrt{5 + \sqrt{13} + 2 x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] - \right. \\ \left. i \sqrt{2} (-65 + 412 \sqrt{13}) x \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\ \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right) / (70 x \sqrt{3 + 5 x^2 + x^4})$$

Problem 167: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2 + 3 x^2) (3 + 5 x^2 + x^4)^{3/2}}{x^4} dx$$

Optimal (type 4, 314 leaves, 5 steps):

$$\frac{949 x (5 + \sqrt{13} + 2 x^2)}{30 \sqrt{3 + 5 x^2 + x^4}} - \frac{13 (24 - 5 x^2) \sqrt{3 + 5 x^2 + x^4}}{15 x} - \\ \frac{(10 - 9 x^2) (3 + 5 x^2 + x^4)^{3/2}}{15 x^3} - \frac{1}{30 \sqrt{3 + 5 x^2 + x^4}} 949 \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} \\ (6 + (5 + \sqrt{13}) x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right] + \\ \frac{1}{\sqrt{3 + 5 x^2 + x^4}} 65 \sqrt{\frac{2}{3 (5 + \sqrt{13})}} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \\ \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right]$$

Result (type 4, 247 leaves):

$$\left(4 (-90 - 1155 x^2 - 1405 x^4 + 192 x^6 + 145 x^8 + 9 x^{10}) + 949 i \sqrt{2} (-5 + \sqrt{13}) x^3 \right. \\
 \left. \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] - \right. \\
 \left. 13 i \sqrt{2} (-65 + 73 \sqrt{13}) x^3 \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\
 \left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right) / \left(60 x^3 \sqrt{3 + 5 x^2 + x^4} \right)$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2 + 3 x^2) (3 + 5 x^2 + x^4)^{3/2}}{x^6} dx$$

Optimal (type 4, 331 leaves, 6 steps):

$$\frac{361 x (5 + \sqrt{13} + 2 x^2)}{15 \sqrt{3 + 5 x^2 + x^4}} - \frac{722 \sqrt{3 + 5 x^2 + x^4}}{15 x} - \frac{(40 - 87 x^2) \sqrt{3 + 5 x^2 + x^4}}{5 x^3} - \\
 \frac{(2 - 5 x^2) (3 + 5 x^2 + x^4)^{3/2}}{5 x^5} - \frac{1}{15 \sqrt{3 + 5 x^2 + x^4}} 361 \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} \\
 (6 + (5 + \sqrt{13}) x^2) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right] + \\
 \left(103 \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \right. \\
 \left. \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x \right], \frac{1}{6} (-13 + 5 \sqrt{13}) \right] \right) / \left(\sqrt{6 (5 + \sqrt{13})} \sqrt{3 + 5 x^2 + x^4} \right)$$

Result (type 4, 244 leaves):

$$\left(-108 - 810 x^2 - 3438 x^4 - 4040 x^6 - 634 x^8 + 30 x^{10} + 361 i \sqrt{2} (-5 + \sqrt{13}) x^5 \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right. \\ \left. \sqrt{5 + \sqrt{13} + 2 x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] - \right. \\ \left. i \sqrt{2} (-260 + 361 \sqrt{13}) x^5 \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\ \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x \right], \frac{19}{6} + \frac{5 \sqrt{13}}{6} \right] \right) / \left(30 x^5 \sqrt{3 + 5 x^2 + x^4} \right)$$

Problem 176: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 (A + B x^2)}{\sqrt{a + b x^2 + c x^4}} dx$$

Optimal (type 4, 403 leaves, 5 steps):

$$- \frac{(4 b B - 5 A c) x \sqrt{a + b x^2 + c x^4}}{15 c^2} + \\ \frac{B x^3 \sqrt{a + b x^2 + c x^4}}{5 c} + \frac{(8 b^2 B - 10 A b c - 9 a B c) x \sqrt{a + b x^2 + c x^4}}{15 c^{5/2} (\sqrt{a} + \sqrt{c} x^2)} - \\ \left(a^{1/4} (8 b^2 B - 10 A b c - 9 a B c) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\ \left. \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(15 c^{11/4} \sqrt{a + b x^2 + c x^4} \right) + \\ \left(a^{1/4} (8 b^2 B - 10 A b c - 9 a B c + \sqrt{a} \sqrt{c} (4 b B - 5 A c)) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\ \left. \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(30 c^{11/4} \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 4, 532 leaves):

$$\begin{aligned}
 & \frac{1}{60 c^3 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}} \\
 & \left(4 c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x (-4 b B+5 A c+3 B c x^2) (a+b x^2+c x^4) + i (8 b^2 B-10 A b c-9 a B c) \right. \\
 & \quad \left. (-b+\sqrt{b^2-4 a c}) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}} \right. \\
 & \quad \text{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right] - \\
 & \quad i\left(-8 b^3 B+b c\left(17 a B-10 A \sqrt{b^2-4 a c}\right)+2 b^2\left(5 A c+4 B \sqrt{b^2-4 a c}\right) - \right. \\
 & \quad \left. a c\left(10 A c+9 B \sqrt{b^2-4 a c}\right)\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}} \\
 & \quad \left. \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right] \right)
 \end{aligned}$$

Problem 177: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (A+B x^2)}{\sqrt{a+b x^2+c x^4}} dx$$

Optimal (type 4, 336 leaves, 4 steps):

$$\frac{B x \sqrt{a+b x^2+c x^4}}{3 c}-\frac{(2 b B-3 A c) x \sqrt{a+b x^2+c x^4}}{3 c^{3 / 2}\left(\sqrt{a}+\sqrt{c} x^2\right)}+\left(a^{1 / 4}(2 b B-3 A c)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right) \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1 / 4} x}{a^{1 / 4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] / \left(3 c^{7 / 4} \sqrt{a+b x^2+c x^4}\right)-\left(a^{1 / 4}\left(2 b B+\sqrt{a} B \sqrt{c}-3 A c\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1 / 4} x}{a^{1 / 4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] / \left(6 c^{7 / 4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 479 leaves):

$$\frac{1}{12 c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}} \sqrt{a+b x^2+c x^4} \left(4 B c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\left(a+b x^2+c x^4\right)-i(2 b B-3 A c)\left(-b+\sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\right) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+i\left(-2 b^2 B+3 A b c+2 a B c+2 b B \sqrt{b^2-4 a c}-3 A c \sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]$$

Problem 178: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\sqrt{a+b x^2+c x^4}} d x$$

Optimal (type 4, 283 leaves, 3 steps):

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}(\sqrt{a}+\sqrt{c}x^2)} - \left(a^{1/4} B (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(c^{3/4} \sqrt{a+bx^2+cx^4} \right) + \left(a^{1/4} \left(B + \frac{A\sqrt{c}}{\sqrt{a}} \right) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(2 c^{3/4} \sqrt{a+bx^2+cx^4} \right)$$

Result (type 4, 302 leaves):

$$\left(i \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \left(B (-b + \sqrt{b^2 - 4ac}) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] + (bB - 2Ac - B\sqrt{b^2 - 4ac}) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] \right) \right) / \left(2\sqrt{2}c \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \sqrt{a+bx^2+cx^4} \right)$$

Problem 179: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + Bx^2}{x^2 \sqrt{a + bx^2 + cx^4}} dx$$

Optimal (type 4, 312 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a(\sqrt{a}+\sqrt{c}x^2)} - \\
 & \left(A c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \\
 & \left(a^{3/4} \sqrt{a+bx^2+cx^4} \right) + \left((\sqrt{a} B + A \sqrt{c}) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\
 & \left. \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(2 a^{3/4} c^{1/4} \sqrt{a+bx^2+cx^4} \right)
 \end{aligned}$$

Result (type 4, 448 leaves):

$$\begin{aligned}
 & \left(-4 A \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} (a+bx^2+cx^4) + \right. \\
 & \left. i A (-b+\sqrt{b^2-4ac}) x \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \right. \\
 & \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] - \right. \\
 & \left. i \left(2 a B + A (-b+\sqrt{b^2-4ac}) \right) x \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \right. \\
 & \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] \right) / \\
 & \left(4 a \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x \sqrt{a+bx^2+cx^4} \right)
 \end{aligned}$$

Problem 180: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}} dx$$

Optimal (type 4, 376 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{A \sqrt{a+b x^2+c x^4}}{3 a x^3} + \frac{(2 A b-3 a B) \sqrt{a+b x^2+c x^4}}{3 a^2 x} - \\
 & \frac{(2 A b-3 a B) \sqrt{c} x \sqrt{a+b x^2+c x^4}}{3 a^2 (\sqrt{a}+\sqrt{c} x^2)} + \left((2 A b-3 a B) c^{1/4} (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \right. \\
 & \left. \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(3 a^{7/4} \sqrt{a+b x^2+c x^4}\right) - \\
 & \left((2 A b-3 a B+\sqrt{a} A \sqrt{c}) c^{1/4} (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \right. \\
 & \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(6 a^{7/4} \sqrt{a+b x^2+c x^4}\right)
 \end{aligned}$$

Result (type 4, 373 leaves):

$$\begin{aligned}
 & \left(-\frac{4(a+b x^2+c x^4)(-2 A b x^2+a(A+3 B x^2))}{x^3} + \right. \\
 & \frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}} i \sqrt{2} \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \\
 & \left(-(2 A b-3 a B)(-b+\sqrt{b^2-4 a c}) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \right. \right. \\
 & \left. \left. \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}} \right] + \left(3 a B(b-\sqrt{b^2-4 a c}) + 2 A(-b^2+a c+b \sqrt{b^2-4 a c}) \right) \text{EllipticF}\left[\right. \right. \\
 & \left. \left. i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}} \right] \right) \right) / \left(12 a^2 \sqrt{a+b x^2+c x^4} \right)
 \end{aligned}$$

Problem 189: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4(2+3 x^2)}{\sqrt{3+5 x^2+x^4}} dx$$

Optimal (type 4, 298 leaves, 5 steps):

$$\frac{419 x (5 + \sqrt{13} + 2 x^2)}{30 \sqrt{3 + 5 x^2 + x^4}} - \frac{10}{3} x \sqrt{3 + 5 x^2 + x^4} + \frac{3}{5} x^3 \sqrt{3 + 5 x^2 + x^4} -$$

$$\frac{1}{30 \sqrt{3 + 5 x^2 + x^4}} 419 \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}}$$

$$(6 + (5 + \sqrt{13}) x^2) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \frac{1}{6} (-13 + 5 \sqrt{13})\right] +$$

$$\frac{1}{\sqrt{3 + 5 x^2 + x^4}} 5 \sqrt{\frac{2}{3 (5 + \sqrt{13})}} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2)$$

$$\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \frac{1}{6} (-13 + 5 \sqrt{13})\right]$$

Result (type 4, 229 leaves):

$$\frac{1}{60 \sqrt{3 + 5 x^2 + x^4}}$$

$$\left(4 x (-150 - 223 x^2 - 5 x^4 + 9 x^6) + 419 i \sqrt{2} (-5 + \sqrt{13}) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right.$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] - i \sqrt{2} (-1795 + 419 \sqrt{13})$$

$$\left. \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] \right)$$

Problem 190: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (2 + 3 x^2)}{\sqrt{3 + 5 x^2 + x^4}} dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{4x(5+\sqrt{13}+2x^2)}{\sqrt{3+5x^2+x^4}} + x\sqrt{3+5x^2+x^4} + \frac{1}{\sqrt{3+5x^2+x^4}} 2\sqrt{\frac{2}{3}(5+\sqrt{13})} \sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}} \\
 & (6+(5+\sqrt{13})x^2) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right], \frac{1}{6}(-13+5\sqrt{13})\right] - \\
 & \frac{1}{\sqrt{3+5x^2+x^4}} \sqrt{\frac{3}{2(5+\sqrt{13})}} \sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}} (6+(5+\sqrt{13})x^2) \\
 & \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right], \frac{1}{6}(-13+5\sqrt{13})\right]
 \end{aligned}$$

Result (type 4, 222 leaves):

$$\begin{aligned}
 & \frac{1}{2\sqrt{3+5x^2+x^4}} \left(2x(3+5x^2+x^4) - 4i\sqrt{2}(-5+\sqrt{13}) \sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2x^2} \right. \\
 & \left. \text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right] + i\sqrt{2}(-17+4\sqrt{13}) \right. \\
 & \left. \sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2x^2} \text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right] \right)
 \end{aligned}$$

Problem 191: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}} dx$$

Optimal (type 4, 257 leaves, 3 steps):

$$\frac{3 x \left(5 + \sqrt{13} + 2 x^2\right)}{2 \sqrt{3 + 5 x^2 + x^4}} - \frac{1}{2 \sqrt{3 + 5 x^2 + x^4}} \sqrt{\frac{3}{2} \left(5 + \sqrt{13}\right)} \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}}$$

$$\left(6 + \left(5 + \sqrt{13}\right) x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right] +$$

$$\frac{1}{\sqrt{3 + 5 x^2 + x^4}} \sqrt{\frac{2}{3 \left(5 + \sqrt{13}\right)}} \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}} \left(6 + \left(5 + \sqrt{13}\right) x^2\right)$$

$$\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right]$$

Result (type 4, 159 leaves):

$$\left(i \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right.$$

$$\left. \left(3 \left(-5 + \sqrt{13}\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] + \right.$$

$$\left. \left. \left(11 - 3 \sqrt{13} \right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] \right) \right) / \left(2 \sqrt{2} \sqrt{3 + 5 x^2 + x^4} \right)$$

Problem 192: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3 x^2}{x^2 \sqrt{3 + 5 x^2 + x^4}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\frac{x \left(5 + \sqrt{13} + 2 x^2\right)}{3 \sqrt{3 + 5 x^2 + x^4}} - \frac{2 \sqrt{3 + 5 x^2 + x^4}}{3 x} - \frac{1}{3 \sqrt{3 + 5 x^2 + x^4}} \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}}$$

$$\left(6 + (5 + \sqrt{13}) x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \frac{1}{6} (-13 + 5 \sqrt{13})\right] +$$

$$\frac{1}{\sqrt{3 + 5 x^2 + x^4}} \sqrt{\frac{3}{2 (5 + \sqrt{13})}} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} \left(6 + (5 + \sqrt{13}) x^2\right)$$

$$\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \frac{1}{6} (-13 + 5 \sqrt{13})\right]$$

Result (type 4, 224 leaves):

$$\frac{1}{6 x \sqrt{3 + 5 x^2 + x^4}} \left(-4 (3 + 5 x^2 + x^4) + i \sqrt{2} (-5 + \sqrt{13}) x \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right.$$

$$\left. \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] - i \sqrt{2} (4 + \sqrt{13}) x \right.$$

$$\left. \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] \right)$$

Problem 193: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3 x^2}{x^4 \sqrt{3 + 5 x^2 + x^4}} dx$$

Optimal (type 4, 302 leaves, 5 steps):

$$\frac{7x(5 + \sqrt{13} + 2x^2)}{54\sqrt{3 + 5x^2 + x^4}} - \frac{2\sqrt{3 + 5x^2 + x^4}}{9x^3} - \frac{7\sqrt{3 + 5x^2 + x^4}}{27x} -$$

$$\frac{1}{54\sqrt{3 + 5x^2 + x^4}} 7 \sqrt{\frac{1}{6}(5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13})x^2}{6 + (5 + \sqrt{13})x^2}}$$

$$(6 + (5 + \sqrt{13})x^2) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}(5 + \sqrt{13})}x\right], \frac{1}{6}(-13 + 5\sqrt{13})\right] -$$

$$\frac{1}{9\sqrt{3 + 5x^2 + x^4}} \sqrt{\frac{2}{3(5 + \sqrt{13})}} \sqrt{\frac{6 + (5 - \sqrt{13})x^2}{6 + (5 + \sqrt{13})x^2}} (6 + (5 + \sqrt{13})x^2)$$

$$\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}(5 + \sqrt{13})}x\right], \frac{1}{6}(-13 + 5\sqrt{13})\right]$$

Result (type 4, 237 leaves):

$$\left(-4(18 + 51x^2 + 41x^4 + 7x^6) + 7i\sqrt{2}(-5 + \sqrt{13})x^3 \sqrt{\frac{-5 + \sqrt{13} - 2x^2}{-5 + \sqrt{13}}}\right.$$

$$\sqrt{5 + \sqrt{13} + 2x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}}x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right] -$$

$$i\sqrt{2}(-47 + 7\sqrt{13})x^3 \sqrt{\frac{-5 + \sqrt{13} - 2x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2x^2}$$

$$\left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}}x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right]\right) / (108x^3\sqrt{3 + 5x^2 + x^4})$$

Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4(2 + 3x^2)}{(3 + 5x^2 + x^4)^{3/2}} dx$$

Optimal (type 4, 307 leaves, 5 steps):

$$\begin{aligned}
 & \frac{43 x \left(5 + \sqrt{13} + 2 x^2\right)}{13 \sqrt{3 + 5 x^2 + x^4}} + \frac{x^3 \left(8 + 11 x^2\right)}{13 \sqrt{3 + 5 x^2 + x^4}} - \frac{11}{13} x \sqrt{3 + 5 x^2 + x^4} - \\
 & \frac{1}{13 \sqrt{3 + 5 x^2 + x^4}} 43 \sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}} \\
 & \left(6 + \left(5 + \sqrt{13}\right) x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right] + \\
 & \frac{1}{13 \sqrt{3 + 5 x^2 + x^4}} 11 \sqrt{\frac{3}{2 \left(5 + \sqrt{13}\right)}} \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}} \left(6 + \left(5 + \sqrt{13}\right) x^2\right) \\
 & \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right]
 \end{aligned}$$

Result (type 4, 219 leaves):

$$\begin{aligned}
 & \frac{1}{26 \sqrt{3 + 5 x^2 + x^4}} \left(-2 x \left(33 + 47 x^2\right) + 43 i \sqrt{2} \left(-5 + \sqrt{13}\right) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2}\right. \\
 & \left. \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] - i \sqrt{2} \left(-182 + 43 \sqrt{13}\right)\right. \\
 & \left. \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right]\right)
 \end{aligned}$$

Problem 200: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(2 + 3 x^2\right)}{\left(3 + 5 x^2 + x^4\right)^{3/2}} dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$\begin{aligned}
& -\frac{11x(5+\sqrt{13}+2x^2)}{26\sqrt{3+5x^2+x^4}} + \frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{1}{26\sqrt{3+5x^2+x^4}} 11\sqrt{\frac{1}{6}(5+\sqrt{13})} \sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}} \\
& (6+(5+\sqrt{13})x^2) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right], \frac{1}{6}(-13+5\sqrt{13})\right] - \\
& \frac{1}{13\sqrt{3+5x^2+x^4}} 4\sqrt{\frac{2}{3(5+\sqrt{13})}} \sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}} (6+(5+\sqrt{13})x^2) \\
& \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right], \frac{1}{6}(-13+5\sqrt{13})\right]
\end{aligned}$$

Result (type 4, 219 leaves):

$$\begin{aligned}
& \frac{1}{52\sqrt{3+5x^2+x^4}} \left(4x(8+11x^2) - 11i\sqrt{2}(-5+\sqrt{13}) \sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2x^2} \right. \\
& \left. \text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right] + i\sqrt{2}(-39+11\sqrt{13}) \right. \\
& \left. \sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2x^2} \text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}x\right], \frac{19}{6} + \frac{5\sqrt{13}}{6}\right] \right)
\end{aligned}$$

Problem 201: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}} dx$$

Optimal (type 4, 282 leaves, 4 steps):

$$\frac{4 x \left(5 + \sqrt{13} + 2 x^2\right)}{39 \sqrt{3 + 5 x^2 + x^4}} - \frac{x \left(7 + 8 x^2\right)}{39 \sqrt{3 + 5 x^2 + x^4}} - \frac{1}{39 \sqrt{3 + 5 x^2 + x^4}} 2 \sqrt{\frac{2}{3} \left(5 + \sqrt{13}\right)} \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}}$$

$$\left(6 + \left(5 + \sqrt{13}\right) x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right] +$$

$$\left(11 \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}} \left(6 + \left(5 + \sqrt{13}\right) x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right] \right) / \left(13 \sqrt{6 \left(5 + \sqrt{13}\right)} \sqrt{3 + 5 x^2 + x^4}\right)$$

Result (type 4, 219 leaves):

$$\frac{1}{78 \sqrt{3 + 5 x^2 + x^4}} \left(-2 x \left(7 + 8 x^2\right) + 4 i \sqrt{2} \left(-5 + \sqrt{13}\right) \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2}\right)$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] - i \sqrt{2} \left(13 + 4 \sqrt{13}\right)$$

$$\sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right]$$

Problem 202: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3 x^2}{x^2 \left(3 + 5 x^2 + x^4\right)^{3/2}} dx$$

Optimal (type 4, 309 leaves, 5 steps):

$$\frac{19 x \left(5 + \sqrt{13} + 2 x^2\right)}{234 \sqrt{3 + 5 x^2 + x^4}} - \frac{7 + 8 x^2}{39 x \sqrt{3 + 5 x^2 + x^4}} - \frac{19 \sqrt{3 + 5 x^2 + x^4}}{117 x} -$$

$$\left(19 \sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}} \left(6 + \left(5 + \sqrt{13}\right) x^2\right) \right.$$

$$\left. \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right]\right) / \left(234 \sqrt{3 + 5 x^2 + x^4}\right) -$$

$$\frac{1}{39 \sqrt{3 + 5 x^2 + x^4}} 4 \sqrt{\frac{2}{3 \left(5 + \sqrt{13}\right)}} \sqrt{\frac{6 + \left(5 - \sqrt{13}\right) x^2}{6 + \left(5 + \sqrt{13}\right) x^2}} \left(6 + \left(5 + \sqrt{13}\right) x^2\right)$$

$$\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5 + \sqrt{13}\right)} x\right], \frac{1}{6} \left(-13 + 5 \sqrt{13}\right)\right]$$

Result (type 4, 228 leaves):

$$\left(-4 \left(78 + 119 x^2 + 19 x^4\right) + 19 i \sqrt{2} \left(-5 + \sqrt{13}\right) x \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}}\right.$$

$$\left. \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] - \right.$$

$$\left. i \sqrt{2} \left(-143 + 19 \sqrt{13}\right) x \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2}\right.$$

$$\left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right]\right) / \left(468 x \sqrt{3 + 5 x^2 + x^4}\right)$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3 x^2}{x^4 \left(3 + 5 x^2 + x^4\right)^{3/2}} dx$$

Optimal (type 4, 326 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{133 x \left(5 + \sqrt{13} + 2 x^2\right)}{1053 \sqrt{3 + 5 x^2 + x^4}} - \frac{7 + 8 x^2}{39 x^3 \sqrt{3 + 5 x^2 + x^4}} - \frac{5 \sqrt{3 + 5 x^2 + x^4}}{351 x^3} + \\
 & \frac{266 \sqrt{3 + 5 x^2 + x^4}}{1053 x} + \left(133 \sqrt{\frac{1}{6} (5 + \sqrt{13})} \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \right. \\
 & \left. \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \frac{1}{6} (-13 + 5 \sqrt{13})\right]\right) / \left(1053 \sqrt{3 + 5 x^2 + x^4}\right) - \\
 & \left(5 \sqrt{\frac{6 + (5 - \sqrt{13}) x^2}{6 + (5 + \sqrt{13}) x^2}} (6 + (5 + \sqrt{13}) x^2) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} (5 + \sqrt{13})} x\right], \right. \right. \\
 & \left. \left. \frac{1}{6} (-13 + 5 \sqrt{13})\right]\right) / \left(351 \sqrt{6 (5 + \sqrt{13})} \sqrt{3 + 5 x^2 + x^4}\right)
 \end{aligned}$$

Result (type 4, 234 leaves):

$$\begin{aligned}
 & \left(-468 + 1014 x^2 + 2630 x^4 + 532 x^6 - 133 i \sqrt{2} (-5 + \sqrt{13}) x^3 \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \right. \\
 & \left. \sqrt{5 + \sqrt{13} + 2 x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right] + \right. \\
 & \left. i \sqrt{2} (-650 + 133 \sqrt{13}) x^3 \sqrt{\frac{-5 + \sqrt{13} - 2 x^2}{-5 + \sqrt{13}}} \sqrt{5 + \sqrt{13} + 2 x^2} \right. \\
 & \left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5 + \sqrt{13}}} x\right], \frac{19}{6} + \frac{5 \sqrt{13}}{6}\right]\right) / \left(2106 x^3 \sqrt{3 + 5 x^2 + x^4}\right)
 \end{aligned}$$

Problem 204: Result more than twice size of optimal antiderivative.

$$\int (f x)^{3/2} (d + e x^2) \sqrt{a + b x^2 + c x^4} dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\left(2 d (f x)^{5/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(5 f \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) +$$

$$\left(2 e (f x)^{9/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{9}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(9 f^3 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right)$$

Result (type 6, 2835 leaves):

$$\frac{1}{x^{3/2}} (f x)^{3/2} \sqrt{a+b x^2+c x^4} \left(\frac{4 (13 b c d-7 b^2 e+18 a c e) \sqrt{x}}{585 c^2} + \frac{2 (13 c d+2 b e) x^{5/2}}{117 c} + \frac{2}{13} e x^{9/2} \right) +$$

$$\left(4 a^3 b d (f x)^{3/2} \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(9 c \left(b-\sqrt{b^2-4 a c} \right) \left(b+\sqrt{b^2-4 a c} \right) x \left(a+b x^2+c x^4 \right)^{3/2} \right.$$

$$\left(-5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) -$$

$$\left(28 a^3 b^2 e (f x)^{3/2} \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(117 c^2 \left(b-\sqrt{b^2-4 a c} \right) \left(b+\sqrt{b^2-4 a c} \right) x \left(a+b x^2+c x^4 \right)^{3/2} \right.$$

$$\left(-5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) +$$

$$\left(8 a^4 e (f x)^{3/2} \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right)$$

$$\begin{aligned}
 & \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \\
 & \left(13 c\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right) x\left(a+b x^2+c x^4\right)^{3 / 2}\right. \\
 & \left(-5 a \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \\
 & \left. x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
 & \left. \left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) - \\
 & \left(8 a^3 d x(f x)^{3 / 2}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\right. \\
 & \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \right. \\
 & \left. \left(5\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3 / 2}\right.\right. \\
 & \left. \left(-9 a \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
 & \left. \left. x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
 & \left. \left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) + \\
 & \left(12 a^2 b^2 d x(f x)^{3 / 2}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\right. \\
 & \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \right. \\
 & \left. \left(25 c\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3 / 2}\right.\right. \\
 & \left. \left(-9 a \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
 & \left. \left. x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
 & \left. \left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) - \\
 & \left(84 a^2 b^3 e x(f x)^{3 / 2}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\right. \\
 & \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \right. \\
 & \left. \left(325 c^2\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3 / 2}\right.\right.
 \end{aligned}$$

$$\begin{aligned} & \left(-9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \Bigg) + \\ & \left(316 a^3 b e x (f x)^{3/2} \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \\ & \quad \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\ & \left(325 c \left(b-\sqrt{b^2-4 a c} \right) \left(b+\sqrt{b^2-4 a c} \right) \left(a+b x^2+c x^4 \right)^{3/2} \right. \\ & \quad \left(-9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \Bigg) \Bigg) \end{aligned}$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int \sqrt{f x} (d+e x^2) \sqrt{a+b x^2+c x^4} dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\begin{aligned} & \left(2 d (f x)^{3/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) / \\ & \left(3 f \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) + \\ & \left(2 e (f x)^{7/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) / \\ & \left(7 f^3 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) \end{aligned}$$

Result (type 6, 1717 leaves):

$$\begin{aligned} & \frac{1}{1617 c^2 (a+b x^2+c x^4)^{3/2}} \\ & x \sqrt{f x} \left(42 c (11 c d+2 b e+7 c e x^2) (a+b x^2+c x^4)^2 + \left(1078 a^2 c d \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] / \\
 & \left(7a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) - \\
 & \left(147a^2be \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(7a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) + \\
 & \left(363abcdx^2 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(11a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) + \\
 & \left(462a^2cex^2 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(11a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) + \\
 & \left(165ab^2ex^2 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right)
 \end{aligned}$$

$$\begin{aligned} & \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \\ & \left(-11 a \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \left. \right) \end{aligned}$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int \frac{(d+e x^2) \sqrt{a+b x^2+c x^4}}{\sqrt{f x}} dx$$

Optimal (type 6, 295 leaves, 6 steps):

$$\begin{aligned} & \left(2 d \sqrt{f x} \sqrt{a+b x^2+c x^4} \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] / \right. \\ & \left(f \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\right) + \\ & \left(2 e (f x)^{5/2} \sqrt{a+b x^2+c x^4} \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] / \right. \\ & \left. \left(5 f^3 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\right) \right) \end{aligned}$$

Result (type 6, 1717 leaves):

$$\begin{aligned} & \frac{1}{225 c^2 \sqrt{f x} (a+b x^2+c x^4)^{3/2}} \\ & x \left(10 c (9 c d+2 b e+5 c e x^2) (a+b x^2+c x^4)^2 + \left(450 a^2 c d (b-\sqrt{b^2-4 a c}+2 c x^2) \right. \right. \\ & \left. \left.(b+\sqrt{b^2-4 a c}+2 c x^2)\right) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \right. \\ & \left(5 a \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \left. \right) - \\ & \left(25 a^2 b e (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right) \end{aligned}$$

$$\begin{aligned}
 & \left. \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\
 & \left(5 a \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\
 & \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) + \\
 & \left(81 a b c d x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\
 & \left(9 a \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\
 & \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) + \\
 & \left(90 a^2 c e x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\
 & \left(9 a \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\
 & \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) + \\
 & \left(27 a b^2 e x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\
 & \left(-9 a \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int \frac{(d+e x^2) \sqrt{a+b x^2+c x^4}}{(f x)^{3/2}} dx$$

Optimal (type 6, 295 leaves, 6 steps):

$$\begin{aligned} & - \left(\left(2 d \sqrt{a+b x^2+c x^4} \operatorname{AppellF1} \left[-\frac{1}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{3}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}} \right] \right) / \right. \\ & \left. \left(f \sqrt{f x} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) \right) + \\ & \left(2 e (f x)^{3/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}} \right] \right) / \\ & \left(3 f^3 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) \end{aligned}$$

Result (type 6, 1383 leaves):

$$\begin{aligned} & \frac{1}{147 (f x)^{3/2} (a+b x^2+c x^4)^{3/2}} \\ & x \left(42 (-7 d+e x^2) (a+b x^2+c x^4)^2 + \left(343 a b d x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \right. \\ & \left. \left. \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \right. \\ & \left(c \left(7 a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] - \right. \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right) \right) + \\ & \left(98 a^2 e x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \right. \right. \\ & \left. \left. \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \right. \\ & \left(c \left(7 a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] - \right. \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
 & \left(462 a d x^4 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(11 a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(33 a b e x^4 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(c \left(11 a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \right) \right)
 \end{aligned}$$

Problem 208: Result more than twice size of optimal antiderivative.

$$\int (f x)^{3/2} (d + e x^2) (a + b x^2 + c x^4)^{3/2} dx$$

Optimal (type 6, 299 leaves, 6 steps):

$$\begin{aligned}
 & \left(2 a d (f x)^{5/2} \sqrt{a + b x^2 + c x^4} \text{AppellF1} \left[\frac{5}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(5 f \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right) + \\
 & \left(2 a e (f x)^{9/2} \sqrt{a + b x^2 + c x^4} \text{AppellF1} \left[\frac{9}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(9 f^3 \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right)
 \end{aligned}$$

Result (type 6, 4499 leaves):

$$\frac{1}{x^{3/2}} (f x)^{3/2} \sqrt{a + b x^2 + c x^4} \left(\frac{8 (-147 b^3 c d + 924 a b c^2 d + 77 b^4 e - 501 a b^2 c e + 612 a^2 c^2 e) \sqrt{x}}{69 615 c^3} + \right.$$

$$\begin{aligned}
 & \frac{2 (84 b^2 c d + 1911 a c^2 d - 44 b^3 e + 240 a b c e) x^{5/2}}{13923 c^2} + \\
 & \frac{2 (399 b c d + 12 b^2 e + 425 a c e) x^{9/2}}{4641 c} + \frac{2}{357} (21 c d + 23 b e) x^{13/2} + \frac{2}{21} c e x^{17/2} \Big) - \\
 & \left(56 a^3 b^3 d (f x)^{3/2} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(663 c^2 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) x \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \Big) \Big) + \\
 & \left(352 a^4 b d (f x)^{3/2} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(663 c \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) x \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \Big) \Big) + \\
 & \left(88 a^3 b^4 e (f x)^{3/2} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(1989 c^3 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) x \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \Big) \Big) -
 \end{aligned}$$

$$\begin{aligned}
 & \left(1336 a^4 b^2 e (fx)^{3/2} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(4641 c^2 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) x (a + bx^2 + cx^4)^{3/2} \right. \\
 & \quad \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) + \\
 & \left(32 a^5 e (fx)^{3/2} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(91 c \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) x (a + bx^2 + cx^4)^{3/2} \right. \\
 & \quad \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\
 & \left(96 a^4 d x (fx)^{3/2} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(85 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) (a + bx^2 + cx^4)^{3/2} \right. \\
 & \quad \left(-9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\
 & \left(504 a^2 b^4 d x (fx)^{3/2} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left(5525 c^2 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(3768 a^3 b^2 d x (f x)^{3/2} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(5525 c \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(264 a^2 b^5 e x (f x)^{3/2} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(5525 c^3 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) - \\
 & \left(2032 a^3 b^3 e x (f x)^{3/2} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(5525 c^2 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \Bigg) + \\
 & \left(26688 a^4 b e x (f x)^{3/2} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(38675 c \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) (a + b x^2 + c x^4)^{3/2} \right. \\
 & \left. \left(-9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\
 & \left. \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \right. \\
 & \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \Bigg) \Bigg)
 \end{aligned}$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int \sqrt{f x} (d + e x^2) (a + b x^2 + c x^4)^{3/2} dx$$

Optimal (type 6, 299 leaves, 6 steps):

$$\begin{aligned}
 & \left(2 a d (f x)^{3/2} \sqrt{a + b x^2 + c x^4} \text{AppellF1} \left[\frac{3}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{7}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(3 f \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right) + \\
 & \left(2 a e (f x)^{7/2} \sqrt{a + b x^2 + c x^4} \text{AppellF1} \left[\frac{7}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(7 f^3 \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right)
 \end{aligned}$$

Result (type 6, 3656 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{x}} \sqrt{f x} \sqrt{a + b x^2 + c x^4} \left(\frac{2 (228 b^2 c d + 3971 a c^2 d - 108 b^3 e + 624 a b c e) x^{3/2}}{21945 c^2} + \right. \\
 & \left. \frac{2 (323 b c d + 12 b^2 e + 345 a c e) x^{7/2}}{3135 c} + \frac{2}{285} (19 c d + 21 b e) x^{11/2} + \frac{2}{19} c e x^{15/2} \right) - \\
 & \left(32 a^4 d x \sqrt{f x} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \\
& \left(15\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3 / 2}\right. \\
& \left(-7 a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \\
& \left. x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
& \left. \left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) + \\
& \left(8 a^3 b^2 d x \sqrt{f x}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\right. \\
& \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \right. \\
& \left. \left(55 c\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3 / 2}\right.\right. \\
& \left. \left(-7 a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
& \left. \left. x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
& \left. \left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)\right) - \\
& \left(72 a^3 b^3 e x \sqrt{f x}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\right. \\
& \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \right. \\
& \left. \left(1045 c^2\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3 / 2}\right.\right. \\
& \left. \left(-7 a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
& \left. \left. x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+ \right. \right. \\
& \left. \left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)\right) + \\
& \left(416 a^4 b e x \sqrt{f x}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\right. \\
& \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \right. \\
& \left. \left(1045 c\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3 / 2}\right.\right.
\end{aligned}$$

$$\begin{aligned}
 & \left(-7 a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \left. \right) - \\
 & \left(288 a^3 b d x^3 \sqrt{f x} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(245 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \left. \right) \left. \right) + \\
 & \left(8 a^2 b^3 d x^3 \sqrt{f x} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(49 c \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \left. \right) \left. \right) - \\
 & \left(96 a^4 e x^3 \sqrt{f x} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(133 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left((b - \sqrt{b^2 - 4ac}) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \Bigg) - \\
 & \left(72 a^2 b^4 e x^3 \sqrt{fx} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(931 c^2 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \Bigg) + \\
 & \left(2472 a^3 b^2 e x^3 \sqrt{fx} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(4655 c \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \Bigg)
 \end{aligned}$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int \frac{(d + e x^2) (a + b x^2 + c x^4)^{3/2}}{\sqrt{fx}} dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\left(2 a d \sqrt{f x} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{5}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(f \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) +$$

$$\left(2 a e (f x)^{5/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(5 f^3 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right)$$

Result (type 6, 3656 leaves):

$$\frac{1}{\sqrt{f x}} \sqrt{x} \sqrt{a+b x^2+c x^4} \left(\frac{2 (68 b^2 c d+867 a c^2 d-28 b^3 e+176 a b c e) \sqrt{x}}{3315 c^2} + \right.$$

$$\left. \frac{2 (85 b c d+4 b^2 e+91 a c e) x^{5/2}}{663 c} + \frac{2}{221} (17 c d+19 b e) x^{9/2} + \frac{2}{17} c e x^{13/2} \right) -$$

$$\left(96 a^4 d x \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(13 \left(b-\sqrt{b^2-4 a c} \right) \left(b+\sqrt{b^2-4 a c} \right) \sqrt{f x} \left(a+b x^2+c x^4 \right)^{3/2} \right.$$

$$\left(-5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) +$$

$$\left(8 a^3 b^2 d x \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(39 c \left(b-\sqrt{b^2-4 a c} \right) \left(b+\sqrt{b^2-4 a c} \right) \sqrt{f x} \left(a+b x^2+c x^4 \right)^{3/2} \right.$$

$$\left(-5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) -$$

$$\begin{aligned}
& \left(56 a^3 b^3 e x \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
& \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
& \left(663 c^2 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \sqrt{f x} \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
& \quad \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
& \left(352 a^4 b e x \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
& \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
& \left(663 c \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \sqrt{f x} \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
& \quad \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) - \\
& \left(672 a^3 b d x^3 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
& \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
& \left(325 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \sqrt{f x} \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
& \quad \left(-9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
& \left(72 a^2 b^3 d x^3 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
& \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) /
\end{aligned}$$

$$\begin{aligned}
 & \left(325 c \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \sqrt{fx} \left(a + bx^2 + cx^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\
 & \left(96 a^4 e x^3 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(85 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \sqrt{fx} \left(a + bx^2 + cx^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\
 & \left(504 a^2 b^4 e x^3 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(5525 c^2 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \sqrt{fx} \left(a + bx^2 + cx^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) + \\
 & \left(3768 a^3 b^2 e x^3 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(5525 c \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \sqrt{fx} \left(a + bx^2 + cx^4 \right)^{3/2} \right. \\
 & \quad \left(-9 a \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right.
 \end{aligned}$$

$$x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right)$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int \frac{(d+ex^2)(ax^2+cx^4)^{3/2}}{(fx)^{3/2}} dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\begin{aligned} & - \left(\left(2ad \sqrt{a+bx^2+cx^4} \text{AppellF1} \left[-\frac{1}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{3}{4}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}} \right] \right) / \right. \\ & \left. \left(f \sqrt{fx} \sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}} \right) \right) + \\ & \left(2ae (fx)^{3/2} \sqrt{a+bx^2+cx^4} \text{AppellF1} \left[\frac{3}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{7}{4}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}} \right] \right) / \\ & \left(3f^3 \sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}} \right) \end{aligned}$$

Result (type 6, 2839 leaves):

$$\begin{aligned} & \frac{1}{(fx)^{3/2}} x^{3/2} \sqrt{a+bx^2+cx^4} \\ & \left(-\frac{2ad}{\sqrt{x}} + \frac{2(195bcd + 12b^2e + 209ace)x^{3/2}}{1155c} + \frac{2}{165} (15cd + 17be)x^{7/2} + \frac{2}{15} cex^{11/2} \right) - \\ & \left(128a^3bdx^3 (b - \sqrt{b^2 - 4ac} + 2cx^2) (b + \sqrt{b^2 - 4ac} + 2cx^2) \right. \\ & \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\ & \left(11 (b - \sqrt{b^2 - 4ac}) (b + \sqrt{b^2 - 4ac}) (fx)^{3/2} (a + bx^2 + cx^4)^{3/2} \right. \\ & \left(-7a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\ & \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\ & \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \end{aligned}$$

$$\begin{aligned}
 & \left(32 a^4 e x^3 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(15 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) (f x)^{3/2} (a + b x^2 + c x^4)^{3/2} \right. \\
 & \quad \left(-7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(8 a^3 b^2 e x^3 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(55 c \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) (f x)^{3/2} (a + b x^2 + c x^4)^{3/2} \right. \\
 & \quad \left(-7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) - \\
 & \left(24 a^2 b^2 d x^5 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(49 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) (f x)^{3/2} (a + b x^2 + c x^4)^{3/2} \right. \\
 & \quad \left(-11 a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) - \\
 & \left(96 a^3 c d x^5 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) /
 \end{aligned}$$

$$\begin{aligned}
& \left(7 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) (f x)^{3/2} (a + b x^2 + c x^4)^{3/2} \right. \\
& \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
& \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
& \quad \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\
& \quad \left(288 a^3 b e x^5 \left(b - \sqrt{b^2 - 4ac} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2 c x^2 \right) \right. \\
& \quad \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
& \quad \left(245 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) (f x)^{3/2} (a + b x^2 + c x^4)^{3/2} \right. \\
& \quad \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
& \quad \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
& \quad \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) \right) + \\
& \quad \left(8 a^2 b^3 e x^5 \left(b - \sqrt{b^2 - 4ac} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2 c x^2 \right) \right. \\
& \quad \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
& \quad \left(49 c \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) (f x)^{3/2} (a + b x^2 + c x^4)^{3/2} \right. \\
& \quad \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
& \quad \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
& \quad \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4ac}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^{3/2} (d + e x^2)}{\sqrt{a + b x^2 + c x^4}} dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\left(2d (fx)^{5/2} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \left(5f \sqrt{a + bx^2 + cx^4} \right) + \\ \left(2e (fx)^{9/2} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ \left. \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \left(9f^3 \sqrt{a + bx^2 + cx^4} \right)$$

Result(type 6, 1037 leaves):

$$\begin{aligned}
& \frac{1}{50 c^2 (a+b x^2+c x^4)^{3/2}} \\
& f \sqrt{f x} \left(20 c e (a+b x^2+c x^4)^2 + \left(25 a^2 e \left(-b+\sqrt{b^2-4 a c}-2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \right. \\
& \quad \left(5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] - \right. \\
& \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \\
& \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right) + \\
& \quad \left(45 a c d x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \\
& \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \\
& \quad \left(9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] - \right. \\
& \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \\
& \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right) + \\
& \quad \left(27 a b e x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \\
& \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \\
& \quad \left(-9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \\
& \quad x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \\
& \quad \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{f x} (d+e x^2)}{\sqrt{a+b x^2+c x^4}} dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\left(2d (fx)^{3/2} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right] \right) / \left(3f \sqrt{a + bx^2 + cx^4} \right) + \\ \left(2e (fx)^{7/2} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ \left. \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right] \right) / \left(7f^3 \sqrt{a + bx^2 + cx^4} \right)$$

Result(type 6, 642 leaves):

$$\frac{1}{42c (a + bx^2 + cx^4)^{3/2}} ax \sqrt{fx} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \\ \left(- \left(\left(49d \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) / \right. \right. \\ \left(-7a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right. \\ \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right. \right. \\ \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right) \right) - \\ \left(33e x^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) / \\ \left(-11a \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right. \\ \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right. \right. \\ \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right) \right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{d + ex^2}{\sqrt{fx} \sqrt{a + bx^2 + cx^4}} dx$$

Optimal (type 6, 295 leaves, 6 steps):

$$\frac{1}{f \sqrt{a+b x^2+c x^4}} 2 d \sqrt{f x} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] +$$

$$\left(2 e (f x)^{5/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\right.$$

$$\left.\operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]\right) / \left(5 f^3 \sqrt{a+b x^2+c x^4}\right)$$

Result (type 6, 642 leaves):

$$\frac{1}{10 c \sqrt{f x} (a+b x^2+c x^4)^{3/2}} a x \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right)$$

$$\left(-\left(\left(25 d \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) / \right.\right.$$

$$\left.\left(-5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.\right.$$

$$\left.\left.x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.\right.$$

$$\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) -$$

$$\left(9 e x^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) /$$

$$\left(-9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left.\left.x^2\left(\left(b+\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.\right.$$

$$\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 215: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{(f x)^{3/2} \sqrt{a+b x^2+c x^4}} dx$$

Optimal (type 6, 295 leaves, 6 steps):

$$\begin{aligned}
 & - \left(\left(2 d \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \operatorname{AppellF1} \left[-\frac{1}{4}, \frac{1}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{1}{2}, \frac{3}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right] \right) / \left(f \sqrt{f x} \sqrt{a + b x^2 + c x^4} \right) \right) + \\
 & \left(2 e (f x)^{3/2} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right] \right) / \left(3 f^3 \sqrt{a + b x^2 + c x^4} \right)
 \end{aligned}$$

Result(type 6, 1049 leaves):

$$\begin{aligned}
 & \frac{1}{21 a (f x)^{3/2} (a + b x^2 + c x^4)^{3/2}} \\
 & 2 x \left(-21 d (a + b x^2 + c x^4)^2 + \left(49 a b d x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \right. \\
 & \quad \left. \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \right. \\
 & \quad \left(4 c \left(7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \quad \left(49 a^2 e x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \right. \right. \\
 & \quad \quad \left. \left. \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \quad \left(4 c \left(7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \quad \left(99 a d x^4 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \right. \right. \\
 & \quad \quad \left. \left. \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \quad \left(44 a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \\
 & \quad \quad 4 x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 216: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^{3/2} (d + e x^2)}{(a + b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 6, 303 leaves, 6 steps):

$$\left(2 d (f x)^{5/2} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \\ \left. \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right] \right) / \left(5 a f \sqrt{a + b x^2 + c x^4} \right) + \\ \left(2 e (f x)^{9/2} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \\ \left. \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right] \right) / \left(9 a f^3 \sqrt{a + b x^2 + c x^4} \right)$$

Result(type 6, 1404 leaves):

$$\frac{1}{5 (b^2 - 4 a c) (a + b x^2 + c x^4)^{3/2}} \\ f \sqrt{f x} \left(5 (-b d + 2 a e - 2 c d x^2 + b e x^2) (a + b x^2 + c x^4) + \left(25 a^2 e (-b + \sqrt{b^2 - 4 a c} - 2 c x^2) \right. \right. \\ \left. \left. (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \right. \\ \left(2 c \left(5 a \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \right. \right. \\ \left. \left. x^2 \left((b + \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right. \right. \right. \\ \left. \left. \left. (b - \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) \right) + \\ \left(25 a b d (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \right. \right. \\ \left. \left. \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \\ \left(4 c \left(5 a \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \right. \right. \\ \left. \left. x^2 \left((b + \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right. \right. \right. \\ \left. \left. \left. (b - \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) \right) + \\ \left(9 a d x^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \right. \right. \\ \left. \left. \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) /$$

$$\begin{aligned} & \left(18 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\ & 2 x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) - \\ & \left(9 a b e x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \\ & \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\ & \left(4 c \left(9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) \end{aligned}$$

Problem 217: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{f x} (d+e x^2)}{(a+b x^2+c x^4)^{3/2}} dx$$

Optimal (type 6, 303 leaves, 6 steps):

$$\begin{aligned} & \left(2 d (f x)^{3/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right. \\ & \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) / \left(3 a f \sqrt{a+b x^2+c x^4} \right) + \\ & \left(2 e (f x)^{7/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right. \\ & \left. \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) / \left(7 a f^3 \sqrt{a+b x^2+c x^4} \right) \end{aligned}$$

Result (type 6, 1740 leaves):

$$\begin{aligned} & \frac{1}{84 a (-b^2+4 a c) (a+b x^2+c x^4)^{3/2}} x \sqrt{f x} \\ & \left(84 (a+b x^2+c x^4) (-b^2 d+b (a e-c d x^2)+2 a c (d+e x^2)) + \left(196 a^2 d \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] / \\
 & \left(14a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad 2x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) + \\
 & \left(49a^2 d \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(c \left(7a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\
 & \left(147a^2 be \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(c \left(7a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\
 & \left(99abd x^2 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(-11a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) + \\
 & \left(198a^2 ex^2 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right)
 \end{aligned}$$

$$\begin{aligned} & \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] / \\ & \left(-11 a \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \end{aligned}$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{\sqrt{f x} (a+b x^2+c x^4)^{3/2}} dx$$

Optimal (type 6, 301 leaves, 6 steps):

$$\begin{aligned} & \left(2 d \sqrt{f x} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right. \\ & \left. \text{AppellF1}\left[\frac{1}{4}, \frac{3}{2}, \frac{3}{2}, \frac{5}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] / \left(a f \sqrt{a+b x^2+c x^4} \right) + \right. \\ & \left(2 e (f x)^{5/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right. \\ & \left. \left. \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] / \left(5 a f^3 \sqrt{a+b x^2+c x^4} \right) \right) \end{aligned}$$

Result (type 6, 1740 leaves):

$$\begin{aligned} & \frac{1}{20 a (-b^2+4 a c) \sqrt{f x} (a+b x^2+c x^4)^{3/2}} \\ & x \left(20 (a+b x^2+c x^4) (-b^2 d+b (a e-c d x^2)+2 a c (d+e x^2)) + \left(300 a^2 d \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \right. \\ & \left. \left. \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \right. \\ & \left(10 a \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\ & 2 x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c} \right) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
 & \left(25 a b^2 d \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(c \left(5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) - \\
 & \left(25 a^2 b e \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \right. \right. \\
 & \quad \left. \left. \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(c \left(5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) - \\
 & \left(9 a b d x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \right. \right. \\
 & \quad \left. \left. \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(-9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(18 a^2 e x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(-9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right.
 \end{aligned}$$

$$\left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right)$$

Problem 219: Result more than twice size of optimal antiderivative.

$$\int \frac{d + ex^2}{(fx)^{3/2} (a + bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 6, 301 leaves, 6 steps):

$$\begin{aligned} & - \left(\left(2d \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \text{AppellF1} \left[-\frac{1}{4}, \frac{3}{2}, \frac{3}{2}, \right. \right. \right. \\ & \quad \left. \left. \left. \frac{3}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \left(af \sqrt{fx} \sqrt{a + bx^2 + cx^4} \right) \right) + \\ & \left(2e (fx)^{3/2} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \text{AppellF1} \left[\frac{3}{4}, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \right. \right. \right. \\ & \quad \left. \left. \left. -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \left(3af^3 \sqrt{a + bx^2 + cx^4} \right) \end{aligned}$$

Result (type 6, 2959 leaves):

$$\begin{aligned} & \frac{1}{(fx)^{3/2}} x^{3/2} \sqrt{a + bx^2 + cx^4} \left(-\frac{2d}{a^2 \sqrt{x}} + \right. \\ & \quad \left. (b^3 dx^{3/2} - 3abcdx^{3/2} - ab^2ex^{3/2} + 2a^2cex^{3/2} + b^2cdx^{7/2} - 2a^2cdx^{7/2} - abcex^{7/2}) / \right. \\ & \quad \left. (a^2(-b^2 + 4ac)(a + bx^2 + cx^4)) \right) + \\ & \left(7b^3 dx^3 (b - \sqrt{b^2 - 4ac} + 2cx^2) (b + \sqrt{b^2 - 4ac} + 2cx^2) \right. \\ & \quad \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\ & \left((-b^2 + 4ac) (b - \sqrt{b^2 - 4ac}) (b + \sqrt{b^2 - 4ac}) (fx)^{3/2} (a + bx^2 + cx^4)^{3/2} \right. \\ & \quad \left(-7a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\ & \quad x^2 \left((b + \sqrt{b^2 - 4ac}) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\ & \quad \left. \left. (b - \sqrt{b^2 - 4ac}) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) - \\ & \left(21abcdx^3 (b - \sqrt{b^2 - 4ac} + 2cx^2) (b + \sqrt{b^2 - 4ac} + 2cx^2) \right) \end{aligned}$$

$$\begin{aligned}
 & \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] / \\
 & \left((-b^2+4ac) (b-\sqrt{b^2-4ac}) (b+\sqrt{b^2-4ac}) (fx)^{3/2} (a+bx^2+cx^4)^{3/2} \right. \\
 & \left. \left(-7a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \right. \\
 & \quad x^2 \left((b+\sqrt{b^2-4ac}) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \\
 & \quad \left. \left. (b-\sqrt{b^2-4ac}) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] \right) \right) \right) - \\
 & \left(7ab^2ex^3 (b-\sqrt{b^2-4ac}+2cx^2) (b+\sqrt{b^2-4ac}+2cx^2) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] / \right. \\
 & \quad \left. \left(3(-b^2+4ac) (b-\sqrt{b^2-4ac}) (b+\sqrt{b^2-4ac}) (fx)^{3/2} (a+bx^2+cx^4)^{3/2} \right. \right. \\
 & \quad \left. \left(-7a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \right. \\
 & \quad \quad x^2 \left((b+\sqrt{b^2-4ac}) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \\
 & \quad \quad \left. \left. (b-\sqrt{b^2-4ac}) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] \right) \right) \right) \right) - \\
 & \left(14a^2cex^3 (b-\sqrt{b^2-4ac}+2cx^2) (b+\sqrt{b^2-4ac}+2cx^2) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] / \right. \\
 & \quad \left. \left(3(-b^2+4ac) (b-\sqrt{b^2-4ac}) (b+\sqrt{b^2-4ac}) (fx)^{3/2} (a+bx^2+cx^4)^{3/2} \right. \right. \\
 & \quad \left. \left(-7a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \right. \\
 & \quad \quad x^2 \left((b+\sqrt{b^2-4ac}) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \\
 & \quad \quad \left. \left. (b-\sqrt{b^2-4ac}) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] \right) \right) \right) \right) + \\
 & \left(99b^2cdx^5 (b-\sqrt{b^2-4ac}+2cx^2) (b+\sqrt{b^2-4ac}+2cx^2) \right. \\
 & \quad \left. \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] / \right. \\
 & \quad \left. \left(7(-b^2+4ac) (b-\sqrt{b^2-4ac}) (b+\sqrt{b^2-4ac}) (fx)^{3/2} (a+bx^2+cx^4)^{3/2} \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \left. \right) - \\
& \left(330 a c^2 d x^5 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
& \left(7 \left(-b^2 + 4 a c \right) \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) (f x)^{3/2} \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
& \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \left. \right) - \\
& \left(33 a b c e x^5 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
& \left(7 \left(-b^2 + 4 a c \right) \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) (f x)^{3/2} \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
& \quad \left(-11 a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
& \quad \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \left. \right) \left. \right)
\end{aligned}$$

Problem 223: Result is not expressed in closed-form.

$$\int \frac{(f x)^m (d + e x^2)}{a + b x^2 + c x^4} dx$$

Optimal (type 5, 194 leaves, 3 steps):

$$\frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right) (f x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right]}{(b-\sqrt{b^2-4ac}) f(1+m)} +$$

$$\frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) (f x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]}{(b+\sqrt{b^2-4ac}) f(1+m)}$$

Result (type 7, 316 leaves):

$$\frac{1}{2m} d (f x)^m \text{RootSum}\left[a + b \#1^2 + c \#1^4 \&, \frac{\text{Hypergeometric2F1}\left[-m, -m, 1-m, -\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m}}{b \#1 + 2c \#1^3} \&\right] +$$

$$\left(e (f x)^m \text{RootSum}\left[a + b \#1^2 + c \#1^4 \&, \frac{1}{b \#1 + 2c \#1^3}\right.\right.$$

$$\left.\left. \left(m x^2 + m^2 x^2 + 2m x \#1 + m^2 x \#1 + 2 \text{Hypergeometric2F1}\left[-m, -m, 1-m, -\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1^2 +\right.\right.$$

$$\left.\left. 3m \text{Hypergeometric2F1}\left[-m, -m, 1-m, -\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1^2 + m^2 \text{Hypergeometric2F1}\left[-m, -m, 1-m, -\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1^2 + m \left(\frac{x}{\#1}\right)^{-m} \#1^2\right) \&\right] \Big/ (2m(1+m)(2+m))$$

Problem 224: Result unnecessarily involves higher level functions.

$$\int \frac{(f x)^m (d+e x^2)}{(a+b x^2+c x^4)^2} dx$$

Optimal (type 5, 392 leaves, 4 steps):

$$\frac{(f x)^{1+m} (b^2 d - 2acd - abe + c(bd - 2ae)x^2)}{2a(b^2 - 4ac) f(a + bx^2 + cx^4)} +$$

$$\left(c \left(b \left(4ae + \sqrt{b^2 - 4ac} d(1-m)\right) - 2a \left(\sqrt{b^2 - 4ac} e(1-m) + 2cd(3-m)\right) + b^2(d-dm)\right)\right.$$

$$\left.(f x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right]\right) \Big/$$

$$\left(2a(b^2 - 4ac)^{3/2} \left(b - \sqrt{b^2 - 4ac}\right) f(1+m)\right) -$$

$$\left(c \left(b \left(4ae - \sqrt{b^2 - 4ac} d(1-m)\right) + 2a \left(\sqrt{b^2 - 4ac} e(1-m) - 2cd(3-m)\right) + b^2 d(1-m)\right)\right.$$

$$\left.(f x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]\right) \Big/$$

$$\left(2a(b^2 - 4ac)^{3/2} \left(b + \sqrt{b^2 - 4ac}\right) f(1+m)\right)$$

Result (type 6, 692 leaves):

$$\frac{1}{4 c (3+m) (a+b x^2+c x^4)^3} a x (f x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \left(\left(d(3+m)^2 \operatorname{AppellF1} \left[\frac{1+m}{2}, 2, 2, \frac{3+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \left((1+m) \left(a(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, 2, 2, \frac{3+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] - 2 x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{3+m}{2}, 2, 3, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{3+m}{2}, 3, 2, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right) + \left(e(5+m) x^2 \operatorname{AppellF1} \left[\frac{3+m}{2}, 2, 2, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] / \left(a(5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 2, 2, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] - 2 x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{5+m}{2}, 2, 3, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1} \left[\frac{5+m}{2}, 3, 2, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right)$$

Problem 225: Result more than twice size of optimal antiderivative.

$$\int (f x)^m (d+e x^2) (a+b x^2+c x^4)^{3/2} dx$$

Optimal (type 6, 319 leaves, 6 steps):

$$\left(a d (f x)^{1+m} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{3}{2}, -\frac{3}{2}, \frac{3+m}{2}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}} \right] / \left(f(1+m) \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} + \left(a e (f x)^{3+m} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{3}{2}, -\frac{3}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}} \right] / \left(f^3 (3+m) \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) \right) \right)$$

Result (type 6, 2559 leaves):

$$\begin{aligned}
 & \left(a \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) d \right. \\
 & \quad (3+m) x (fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \\
 & \quad \left. \text{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) / \\
 & \left(8c^2 (1+m) \sqrt{a+bx^2+cx^4} \left(2a (3+m) \text{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, \right. \right. \right. \\
 & \quad \left. \left. -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \right. \right. \\
 & \quad \left. \left. \text{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \left(b - \sqrt{b^2 - 4ac} \right) \right. \right. \\
 & \quad \left. \left. \left. \left. \text{AppellF1} \left[\frac{3+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) \right) \right) + \\
 & \left(b \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) d (5+m) x^3 (fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \\
 & \quad \left. \text{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) / \\
 & \left(8c^2 (3+m) \sqrt{a+bx^2+cx^4} \right. \\
 & \quad \left(2a (5+m) \text{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \\
 & \quad \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \right. \\
 & \quad \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5+m}{2}, \frac{1}{2}, -\frac{1}{2}, \right. \right. \\
 & \quad \left. \left. \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) \right) + \\
 & \left(a \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) e (5+m) x^3 (fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \\
 & \quad \left. \text{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) / \\
 & \left(8c^2 (3+m) \sqrt{a+bx^2+cx^4} \right. \\
 & \quad \left(2a (5+m) \text{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \\
 & \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \Bigg) + \\
 & \left(\left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) d (7+m) x^5 (fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \left. \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \left. \text{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \Bigg) / \\
 & \left(8c(5+m) \sqrt{a+bx^2+cx^4} \right. \\
 & \left. \left(2a(7+m) \text{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \right. \\
 & \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{9+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \right. \\
 & \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \Bigg) \Bigg) + \\
 & \left(b \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) e (7+m) x^5 (fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \left. \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \left. \text{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \Bigg) / \\
 & \left(8c^2(5+m) \sqrt{a+bx^2+cx^4} \right. \\
 & \left. \left(2a(7+m) \text{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \right. \\
 & \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{9+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \right. \\
 & \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \Bigg) \Bigg) + \\
 & \left(\left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) e (9+m) x^7 (fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \left. \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right)
 \end{aligned}$$

$$\begin{aligned} & \text{AppellF1}\left[\frac{7+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] \Big/ \\ & \left(8c(7+m)\sqrt{a+bx^2+cx^4}\right. \\ & \left. \left(2a(9+m)\text{AppellF1}\left[\frac{7+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \right. \\ & \left. \left. x^2\left(\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{11+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + \right. \right. \right. \\ & \left. \left. \left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9+m}{2}, \frac{1}{2}, -\frac{1}{2}, \right. \right. \right. \\ & \left. \left. \left. \frac{11+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right) \end{aligned}$$

Problem 226: Result more than twice size of optimal antiderivative.

$$\int (fx)^m (d+ex^2) \sqrt{a+bx^2+cx^4} dx$$

Optimal (type 6, 317 leaves, 6 steps):

$$\begin{aligned} & \left(d(fx)^{1+m}\sqrt{a+bx^2+cx^4}\text{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]\right) \Big/ \\ & \left(f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right) + \\ & \left(e(fx)^{3+m}\sqrt{a+bx^2+cx^4}\text{AppellF1}\left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, \right. \right. \\ & \left. \left. -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]\right) \Big/ \left(f^3(3+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right) \end{aligned}$$

Result (type 6, 755 leaves):

$$\frac{1}{8 c^2 (3+m) \sqrt{a+b x^2+c x^4}} \left((b-\sqrt{b^2-4 a c}) (b+\sqrt{b^2-4 a c}) x (f x)^m (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right. \\ \left. \left(\left(d (3+m)^2 \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \right. \right. \\ \left. \left((1+m) \left(2 a (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) + \right. \right. \\ \left. x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + (b-\sqrt{b^2-4 a c}) \right. \right. \\ \left. \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right) + \\ \left(e (5+m) x^2 \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) / \\ \left(2 a (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \\ \left. x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] + \right. \right. \\ \left. \left. (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1} \left[\frac{5+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}} \right] \right) \right) \right)$$

Problem 227: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^m (d+e x^2)}{\sqrt{a+b x^2+c x^4}} dx$$

Optimal (type 6, 317 leaves, 6 steps):

$$\left(d (fx)^{1+m} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right] \right) /$$

$$\left(f(1+m) \sqrt{a+bx^2+cx^4} + \left(e (fx)^{3+m} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right] \right) / \left(f^3 (3+m) \sqrt{a+bx^2+cx^4} \right) \right)$$

Result (type 6, 728 leaves):

$$\frac{1}{2c(3+m)(a+bx^2+cx^4)^{3/2}} ax (fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right)$$

$$\left(\left(d(3+m)^2 \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) / \right.$$

$$\left((1+m) \left(2a(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] - \right.$$

$$x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right.$$

$$\left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{3}{2}, \frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right) -$$

$$\left(e(5+m)x^2 \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) /$$

$$\left(-2a(5+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right.$$

$$x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{5+m}{2}, \frac{1}{2}, \frac{3}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right.$$

$$\left. \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{5+m}{2}, \frac{3}{2}, \frac{1}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right)$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}} dx$$

Optimal (type 6, 323 leaves, 6 steps):

$$\left(d (fx)^{1+m} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ \left. \text{AppellF1} \left[\frac{1+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \\ \left(af(1+m) \sqrt{a+bx^2+cx^4} \right) + \left(e (fx)^{3+m} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \text{AppellF1} \left[\right. \right. \\ \left. \left. \frac{3+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \left(af^3(3+m) \sqrt{a+bx^2+cx^4} \right)$$

Result (type 6, 728 leaves):

$$\frac{1}{2c(3+m)(a+bx^2+cx^4)^{5/2}} ax(fx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \\ \left(\left(d(3+m)^2 \text{AppellF1} \left[\frac{1+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \right. \\ \left((1+m) \left(2a(3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \right. \\ \left. \left. 3x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{3+m}{2}, \frac{3}{2}, \frac{5}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \left(b - \sqrt{b^2 - 4ac} \right) \right. \right. \right. \\ \left. \left. \left. \text{AppellF1} \left[\frac{3+m}{2}, \frac{5}{2}, \frac{3}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) + \\ \left(e(5+m)x^2 \text{AppellF1} \left[\frac{3+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\ \left(2a(5+m) \text{AppellF1} \left[\frac{3+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\ \left. 3x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5+m}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\ \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{5+m}{2}, \frac{5}{2}, \frac{3}{2}, \frac{7+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right)$$

Problem 259: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

Optimal (type 4, 70 leaves, 4 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2} x}{\sqrt{1+x^4}}\right]}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 40 leaves):

$$(-1)^{1/4} \left(-\text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] + \text{EllipticPi}\left[-\text{i}, \text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 260: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}} dx$$

Optimal (type 4, 70 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} x}{\sqrt{1+x^4}}\right]}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 36 leaves):

$$(-1)^{1/4} \left(\text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] - \text{EllipticPi}\left[\text{i}, \text{ArcSin}\left[(-1)^{3/4} x\right], -1\right] \right)$$

Problem 265: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}} dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{2} x}{\sqrt{-1-x^4}}\right]}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{4\sqrt{-1-x^4}}$$

Result (type 4, 60 leaves):

$$\frac{1}{\sqrt{-1-x^4}} (-1)^{1/4} \sqrt{1+x^4} \left(-\text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] + \text{EllipticPi}\left[-\text{i}, \text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 266: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}} dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2} x}{\sqrt{-1-x^4}}\right]}{2 \sqrt{2}} - \frac{(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{4 \sqrt{-1-x^4}}$$

Result (type 4, 56 leaves):

$$\frac{1}{\sqrt{-1-x^4}} (-1)^{1/4} \sqrt{1+x^4} \left(\text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right], -1\right] - \text{EllipticPi}\left[i, \text{ArcSin}\left[(-1)^{3/4} x\right], -1\right] \right)$$

Problem 310: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{f x} (d+e x^2) (a+b x^2+c x^4)} dx$$

Optimal (type 3, 866 leaves, 19 steps):

$$\begin{aligned} & \frac{c^{3/4} \left(2 c d - \left(b - \sqrt{b^2 - 4 a c}\right) e\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{f x}}{\left(-b - \sqrt{b^2 - 4 a c}\right)^{1/4} \sqrt{f}}\right]}{2^{1/4} \sqrt{b^2 - 4 a c} \left(-b - \sqrt{b^2 - 4 a c}\right)^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} - \\ & \frac{c^{3/4} \left(2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{f x}}{\left(-b + \sqrt{b^2 - 4 a c}\right)^{1/4} \sqrt{f}}\right]}{2^{1/4} \sqrt{b^2 - 4 a c} \left(-b + \sqrt{b^2 - 4 a c}\right)^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} - \frac{e^{7/4} \text{ArcTan}\left[1 - \frac{\sqrt{2} e^{1/4} \sqrt{f x}}{d^{1/4} \sqrt{f}}\right]}{\sqrt{2} d^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} + \\ & \frac{e^{7/4} \text{ArcTan}\left[1 + \frac{\sqrt{2} e^{1/4} \sqrt{f x}}{d^{1/4} \sqrt{f}}\right]}{\sqrt{2} d^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} + \frac{c^{3/4} \left(2 c d - \left(b - \sqrt{b^2 - 4 a c}\right) e\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{f x}}{\left(-b - \sqrt{b^2 - 4 a c}\right)^{1/4} \sqrt{f}}\right]}{2^{1/4} \sqrt{b^2 - 4 a c} \left(-b - \sqrt{b^2 - 4 a c}\right)^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} - \\ & \frac{c^{3/4} \left(2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{f x}}{\left(-b + \sqrt{b^2 - 4 a c}\right)^{1/4} \sqrt{f}}\right]}{2^{1/4} \sqrt{b^2 - 4 a c} \left(-b + \sqrt{b^2 - 4 a c}\right)^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} - \\ & \frac{e^{7/4} \text{Log}\left[\sqrt{d} \sqrt{f} + \sqrt{e} \sqrt{f} x - \sqrt{2} d^{1/4} e^{1/4} \sqrt{f x}\right]}{2 \sqrt{2} d^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} + \\ & \frac{e^{7/4} \text{Log}\left[\sqrt{d} \sqrt{f} + \sqrt{e} \sqrt{f} x + \sqrt{2} d^{1/4} e^{1/4} \sqrt{f x}\right]}{2 \sqrt{2} d^{3/4} \left(c d^2 - b d e + a e^2\right) \sqrt{f}} \end{aligned}$$

Result (type 7, 267 leaves):

$$\left(\sqrt{x} \left(\sqrt{2} e^{7/4} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} e^{1/4} \sqrt{x}}{d^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} e^{1/4} \sqrt{x}}{d^{1/4}} \right] - \operatorname{Log} \left[\sqrt{d} - \sqrt{2} d^{1/4} e^{1/4} \sqrt{x} + \sqrt{e} x \right] + \operatorname{Log} \left[\sqrt{d} + \sqrt{2} d^{1/4} e^{1/4} \sqrt{x} + \sqrt{e} x \right] - 2 d^{3/4} \operatorname{RootSum} \left[a + b \#1^4 + c \#1^8 \&, \frac{-c d \operatorname{Log}[\sqrt{x} - \#1] + b e \operatorname{Log}[\sqrt{x} - \#1] + c e \operatorname{Log}[\sqrt{x} - \#1] \#1^4}{b \#1^3 + 2 c \#1^7} \& \right] \right) \right) / \left(4 d^{3/4} (c d^2 + e (-b d + a e)) \sqrt{f x} \right)$$

Problem 316: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \sqrt{1+2x^2+2x^4}}{3+2x^2} dx$$

Optimal (type 4, 424 leaves, 17 steps):

$$\begin{aligned} & -\frac{1}{60} x (13 - 6x^2) \sqrt{1+2x^2+2x^4} + \frac{109 x \sqrt{1+2x^2+2x^4}}{60 \sqrt{2} (1 + \sqrt{2} x^2)} + \frac{3}{16} \sqrt{15} \operatorname{ArcTan} \left[\frac{\sqrt{\frac{5}{3} x}}{\sqrt{1+2x^2+2x^4}} \right] - \\ & \left(\frac{109 (1 + \sqrt{2} x^2) \sqrt{\frac{1+2x^2+2x^4}{(1 + \sqrt{2} x^2)^2}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} [2^{1/4} x], \frac{1}{4} (2 - \sqrt{2}) \right] \right) / \\ & \left(60 \times 2^{3/4} \sqrt{1+2x^2+2x^4} \right) + \\ & \left(\frac{(-70 + 263 \sqrt{2}) (1 + \sqrt{2} x^2) \sqrt{\frac{1+2x^2+2x^4}{(1 + \sqrt{2} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} [2^{1/4} x], \frac{1}{4} (2 - \sqrt{2}) \right] \right) / \\ & \left(60 \times 2^{3/4} (-2 + 3 \sqrt{2}) \sqrt{1+2x^2+2x^4} \right) + \\ & \left(\frac{15 (3 + \sqrt{2}) (1 + \sqrt{2} x^2) \sqrt{\frac{1+2x^2+2x^4}{(1 + \sqrt{2} x^2)^2}} \operatorname{EllipticPi} \left[\frac{1}{24} (12 - 11 \sqrt{2}), \right. \right. \\ & \left. \left. 2 \operatorname{ArcTan} [2^{1/4} x], \frac{1}{4} (2 - \sqrt{2}) \right] \right) / \left(16 \times 2^{3/4} (2 - 3 \sqrt{2}) \sqrt{1+2x^2+2x^4} \right) \end{aligned}$$

Result (type 4, 209 leaves):

$$\frac{1}{240 \sqrt{1+2 x^2+2 x^4}} \left(-52 x - 80 x^3 - 56 x^5 + 48 x^7 - \right. \\
218 i \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticE}[i \text{ArcSinh}[\sqrt{1-i} x], i] - \\
(199 - 417 i) \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticF}[i \text{ArcSinh}[\sqrt{1-i} x], i] + \\
\left. 225 (1-i)^{3/2} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i \text{ArcSinh}[\sqrt{1-i} x], i\right] \right)$$

Problem 317: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \sqrt{1+2 x^2+2 x^4}}{3+2 x^2} dx$$

Optimal (type 4, 417 leaves, 13 steps):

$$\frac{1}{6} x \sqrt{1+2 x^2+2 x^4} - \frac{7 x \sqrt{1+2 x^2+2 x^4}}{6 \sqrt{2} (1+\sqrt{2} x^2)} - \frac{1}{8} \sqrt{15} \text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}} x}{\sqrt{1+2 x^2+2 x^4}}\right] + \\
\frac{7 (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]}{6 \times 2^{3/4} \sqrt{1+2 x^2+2 x^4}} - \\
\left((-4+17 \sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \\
\left(6 \times 2^{3/4} (-2+3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4} \right) - \\
\left(5 (3+\sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \text{EllipticPi}\left[\frac{1}{24} (12-11 \sqrt{2}), \right. \right. \\
\left. \left. 2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \left(8 \times 2^{3/4} (2-3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4} \right)$$

Result (type 4, 204 leaves):

$$\left(4 x + 8 x^3 + 8 x^5 + 14 i \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticE}[i \text{ArcSinh}[\sqrt{1-i} x], i] + \right. \\
(13 - 27 i) \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticF}[i \text{ArcSinh}[\sqrt{1-i} x], i] - \\
15 (1-i)^{3/2} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \\
\left. \text{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i \text{ArcSinh}[\sqrt{1-i} x], i\right] \right) / \left(24 \sqrt{1+2 x^2+2 x^4} \right)$$

Problem 318: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2} dx$$

Optimal (type 4, 381 leaves, 7 steps):

$$\begin{aligned} & \frac{x \sqrt{1+2x^2+2x^4}}{\sqrt{2} (1+\sqrt{2}x^2)} + \frac{1}{4} \sqrt{\frac{5}{3}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \\ & \frac{(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]}{2^{3/4} \sqrt{1+2x^2+2x^4}} + \\ & \left(2^{3/4} (1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\ & \left((-2+3\sqrt{2}) \sqrt{1+2x^2+2x^4} \right) + \\ & \left(5(3+\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), \right. \right. \\ & \left. \left. 2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(12 \times 2^{3/4} (2-3\sqrt{2}) \sqrt{1+2x^2+2x^4} \right) \end{aligned}$$

Result (type 4, 127 leaves):

$$\begin{aligned} & - \left(\left(\sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \right. \right. \\ & \left. \left((3+3i) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - (3+6i) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], \right. \right. \right. \\ & \left. \left. \left. i\right] + 5i \operatorname{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] \right) \right) / \left(6 \sqrt{1-i} \sqrt{1+2x^2+2x^4} \right) \end{aligned}$$

Problem 319: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)} dx$$

Optimal (type 4, 399 leaves, 8 steps):

$$\begin{aligned}
& -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3(1+\sqrt{2}x^2)} - \frac{1}{6}\sqrt{\frac{5}{3}}\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \frac{1}{3\sqrt{1+2x^2+2x^4}} \\
& 2^{1/4}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticE}\left[2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] + \\
& \left((3+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticF}\left[2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
& \left(21 \times 2^{1/4}\sqrt{1+2x^2+2x^4} \right) + \left(5(3+\sqrt{2})^2(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \right. \\
& \left. \text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), 2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(252 \times 2^{1/4}\sqrt{1+2x^2+2x^4} \right)
\end{aligned}$$

Result (type 4, 208 leaves):

$$\begin{aligned}
& (-6 - 12x^2 - 12x^4 - \\
& 6i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticE}[i\text{ArcSinh}[\sqrt{1-i}x], i] + \\
& (9-3i)\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}[i\text{ArcSinh}[\sqrt{1-i}x], i] - \\
& 5(1-i)^{3/2}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2} \\
& \text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\text{ArcSinh}[\sqrt{1-i}x], i\right]) / (18x\sqrt{1+2x^2+2x^4})
\end{aligned}$$

Problem 320: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)} dx$$

Optimal (type 4, 360 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \\
 & \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]}{9 \times 2^{1/4} \sqrt{1+2x^2+2x^4}} + \\
 & \left(5(3+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\
 & \left(63 \times 2^{1/4} \sqrt{1+2x^2+2x^4}\right) - \left(5(3+\sqrt{2})^2(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\right. \\
 & \left. \operatorname{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), 2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \left(378 \times 2^{1/4} \sqrt{1+2x^2+2x^4}\right)
 \end{aligned}$$

Result (type 4, 154 leaves):

$$\begin{aligned}
 & -\left(\left(3+6x^2+6x^4+3(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\right.\right. \\
 & \quad \left.\left.\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right]-5(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\right.\right. \\
 & \quad \left.\left.\operatorname{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right]\right)\right) / \left(27x^3\sqrt{1+2x^2+2x^4}\right)
 \end{aligned}$$

Problem 321: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)} dx$$

Optimal (type 4, 546 leaves, 13 steps):

$$\begin{aligned}
 & -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \\
 & \frac{4\sqrt{2}x\sqrt{1+2x^2+2x^4}}{45(1+\sqrt{2}x^2)} - \frac{2}{27}\sqrt{\frac{5}{3}}\operatorname{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \\
 & \left(4 \times 2^{1/4} (1 + \sqrt{2} x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\
 & (45\sqrt{1+2x^2+2x^4}) + \\
 & \left(5 \times 2^{1/4} (5-3\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\
 & (189\sqrt{1+2x^2+2x^4}) - \\
 & \left(2^{1/4} (19-2\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\
 & (135\sqrt{1+2x^2+2x^4}) + \left(5(3+\sqrt{2})^2(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \right. \\
 & \left. \operatorname{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), 2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / (567 \times 2^{1/4} \sqrt{1+2x^2+2x^4})
 \end{aligned}$$

Result (type 4, 224 leaves):

$$\begin{aligned}
 & -\frac{1}{405x^5\sqrt{1+2x^2+2x^4}} \left(27+42x^2+66x^4+48x^6+72x^8 + \right. \\
 & 36i\sqrt{1-i}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \\
 & (12+24i)\sqrt{1-i}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] + \\
 & \left. 50(1-i)^{3/2}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\operatorname{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right]\right)
 \end{aligned}$$

Problem 327: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2} dx$$

Optimal (type 4, 463 leaves, 19 steps):

$$\begin{aligned}
 & -\frac{213}{140} x \sqrt{1+2 x^2+2 x^4} - \frac{27}{70} x^3 \sqrt{1+2 x^2+2 x^4} - \frac{2211 x \sqrt{1+2 x^2+2 x^4}}{140 \sqrt{2} (1+\sqrt{2} x^2)} - \\
 & \frac{1}{14} x (1+2 x^2+2 x^4)^{3/2} + \frac{17}{16} \sqrt{51} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}} x}{\sqrt{1+2 x^2+2 x^4}}\right] + \\
 & \left(\frac{2211 (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]}{\left(140 \times 2^{3/4} \sqrt{1+2 x^2+2 x^4}\right)} - \right. \\
 & \left. \frac{3 (514+2717 \sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]}{\left(140 \times 2^{3/4} (2+3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4}\right)} - \right. \\
 & \left. \frac{289 (3-\sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticPi}\left[\frac{1}{24} (12+11 \sqrt{2}), \right. \right. \\
 & \left. \left. 2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]}{\left(16 \times 2^{3/4} (2+3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4}\right)} \right) /
 \end{aligned}$$

Result (type 4, 214 leaves):

$$\begin{aligned}
 & \frac{1}{560 \sqrt{1+2 x^2+2 x^4}} \left(-892 x - 2080 x^3 - 2456 x^5 - 752 x^7 - 160 x^9 + \right. \\
 & 4422 i \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] - \\
 & (9669 - 5247 i) \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] + \\
 & \left. 10115 (1-i)^{3/2} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \operatorname{EllipticPi}\left[-\frac{1}{3} - \frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] \right)
 \end{aligned}$$

Problem 328: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+2 x^2+2 x^4)^{3/2}}{3-2 x^2} dx$$

Optimal (type 4, 428 leaves, 12 steps):

$$\begin{aligned}
 & -\frac{1}{10} x (9+2 x^2) \sqrt{1+2 x^2+2 x^4} - \frac{103 x \sqrt{1+2 x^2+2 x^4}}{10 \sqrt{2} (1+\sqrt{2} x^2)} + \frac{17}{8} \sqrt{\frac{17}{3}} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}} x}{\sqrt{1+2 x^2+2 x^4}}\right] + \\
 & \left(\frac{103 (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]\right) / \\
 & \left(10 \times 2^{3/4} \sqrt{1+2 x^2+2 x^4} \right) - \\
 & \left(\frac{(66+383 \sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]\right) / \\
 & \left(10 \times 2^{3/4} (2+3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4} \right) - \\
 & \left(\frac{289 (3-\sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticPi}\left[\frac{1}{24} (12+11 \sqrt{2}), \right. \right. \\
 & \left. \left. 2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]\right) / \left(24 \times 2^{3/4} (2+3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4} \right)
 \end{aligned}$$

Result (type 4, 209 leaves):

$$\begin{aligned}
 & \frac{1}{120 \sqrt{1+2 x^2+2 x^4}} \left(-108 x - 240 x^3 - 264 x^5 - 48 x^7 + \right. \\
 & 618 i \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] - \\
 & (1371-753 i) \sqrt{1-i} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] + \\
 & \left. 1445 (1-i)^{3/2} \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \operatorname{EllipticPi}\left[-\frac{1}{3}-\frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] \right)
 \end{aligned}$$

Problem 329: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+2 x^2+2 x^4)^{3/2}}{x^2 (3-2 x^2)} dx$$

Optimal (type 4, 722 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{(1+x^2) \sqrt{1+2x^2+2x^4}}{3x} - \frac{17x \sqrt{1+2x^2+2x^4}}{3\sqrt{2}(1+\sqrt{2}x^2)} + \\
 & \frac{\sqrt{2}x \sqrt{1+2x^2+2x^4}}{3(1+\sqrt{2}x^2)} + \frac{17}{12} \sqrt{\frac{17}{3}} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] + \\
 & \left(17(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\
 & \left(3 \times 2^{3/4} \sqrt{1+2x^2+2x^4}\right) - \frac{1}{3\sqrt{1+2x^2+2x^4}} \\
 & 2^{1/4}(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] + \\
 & \frac{(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]}{3 \times 2^{3/4} \sqrt{1+2x^2+2x^4}} + \\
 & \left(289(3-\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\
 & \left(84 \times 2^{1/4} \sqrt{1+2x^2+2x^4}\right) - \\
 & \left(17(5+\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\
 & \left(12 \times 2^{1/4} \sqrt{1+2x^2+2x^4}\right) - \left(289(11-6\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \right. \\
 & \left. \operatorname{EllipticPi}\left[\frac{1}{24}(12+11\sqrt{2}), 2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \left(504 \times 2^{1/4} \sqrt{1+2x^2+2x^4}\right)
 \end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
 & \frac{1}{36x \sqrt{1+2x^2+2x^4}} \left(-12 - 36x^2 - 48x^4 - 24x^6 + \right. \\
 & 90i \sqrt{1-i} x \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \\
 & (255-165i) \sqrt{1-i} x \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] + \\
 & \left. 289(1-i)^{3/2} x \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \operatorname{EllipticPi}\left[-\frac{1}{3}-\frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] \right)
 \end{aligned}$$

Problem 330: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)} dx$$

Optimal (type 4, 625 leaves, 13 steps):

$$\begin{aligned} & -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} + \\ & \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{9(1+\sqrt{2}x^2)} + \frac{17}{18}\sqrt{\frac{17}{3}}\operatorname{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \frac{1}{9\sqrt{1+2x^2+2x^4}} \\ & 2^{1/4}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] + \\ & \left(289(3-\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\ & (126 \times 2^{1/4}\sqrt{1+2x^2+2x^4}) - \\ & \left(17(5+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right) / \\ & (18 \times 2^{1/4}\sqrt{1+2x^2+2x^4}) + \frac{1}{9\sqrt{1+2x^2+2x^4}} \\ & 2^{1/4}(9+5\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] - \\ & \left(289(11-6\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\right) \\ & \operatorname{EllipticPi}\left[\frac{1}{24}(12+11\sqrt{2}), 2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] / (756 \times 2^{1/4}\sqrt{1+2x^2+2x^4}) \end{aligned}$$

Result (type 4, 219 leaves):

$$\frac{1}{54 x^3 \sqrt{1+2 x^2+2 x^4}} \left(-6 - 72 x^2 - 132 x^4 - 120 x^6 - \right. \\ \left. 6 i \sqrt{1-i} x^3 \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{1-i} x\right], i\right] - \right. \\ \left. (195 - 201 i) \sqrt{1-i} x^3 \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1-i} x\right], i\right] + \right. \\ \left. 289 (1-i)^{3/2} x^3 \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \text{EllipticPi}\left[-\frac{1}{3} - \frac{i}{3}, i \text{ArcSinh}\left[\sqrt{1-i} x\right], i\right] \right)$$

Problem 331: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+2 x^2+2 x^4)^{3/2}}{x^6 (3-2 x^2)} dx$$

Optimal (type 4, 553 leaves, 15 steps):

$$\frac{74 \sqrt{1+2 x^2+2 x^4}}{135 x^3} - \frac{262 \sqrt{1+2 x^2+2 x^4}}{135 x} - \frac{(3+40 x^2) \sqrt{1+2 x^2+2 x^4}}{45 x^5} + \\ \frac{262 \sqrt{2} x \sqrt{1+2 x^2+2 x^4}}{135 (1+\sqrt{2} x^2)} + \frac{17}{27} \sqrt{\frac{17}{3}} \text{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}} x}{\sqrt{1+2 x^2+2 x^4}}\right] - \\ \left(\frac{262 \times 2^{1/4} (1+\sqrt{2} x^2)}{(1+\sqrt{2} x^2)^2} \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \\ (135 \sqrt{1+2 x^2+2 x^4}) + \\ \left(\frac{85 \times 2^{3/4} (3-\sqrt{2}) (1+\sqrt{2} x^2)}{(1+\sqrt{2} x^2)^2} \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \\ (189 \sqrt{1+2 x^2+2 x^4}) + \\ \left(\frac{2^{3/4} (37+23 \sqrt{2}) (1+\sqrt{2} x^2)}{(1+\sqrt{2} x^2)^2} \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \\ (135 \sqrt{1+2 x^2+2 x^4}) - \left(\frac{289 (11-6 \sqrt{2}) (1+\sqrt{2} x^2)}{(1+\sqrt{2} x^2)^2} \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \right. \\ \left. \text{EllipticPi}\left[\frac{1}{24} (12+11 \sqrt{2}), 2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / (1134 \times 2^{1/4} \sqrt{1+2 x^2+2 x^4})$$

Result (type 4, 224 leaves):

$$\begin{aligned}
 & - \frac{1}{405 x^5 \sqrt{1+2 x^2+2 x^4}} \\
 & \left(27 + 192 x^2 + 1116 x^4 + 1848 x^6 + 1572 x^8 + 786 i \sqrt{1-i} x^5 \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \right. \\
 & \quad \text{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] + (543 - 1329 i) \sqrt{1-i} x^5 \sqrt{1+(1-i) x^2} \\
 & \quad \sqrt{1+(1+i) x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] - 1445 (1-i)^{3/2} x^5 \\
 & \quad \left. \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \operatorname{EllipticPi}\left[-\frac{1}{3} - \frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] \right)
 \end{aligned}$$

Problem 337: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{(3+2 x^2) \sqrt{1+2 x^2+2 x^4}} dx$$

Optimal (type 4, 418 leaves, 4 steps):

$$\begin{aligned}
 & \frac{x \sqrt{1+2 x^2+2 x^4}}{2 \sqrt{2} (1+\sqrt{2} x^2)} - \frac{3 \sqrt{\frac{3}{10}} (3-\sqrt{2}) \operatorname{ArcTan}\left[\frac{\sqrt{\frac{5}{3}} x}{\sqrt{1+2 x^2+2 x^4}}\right]}{4 (2-3 \sqrt{2})} \\
 & \frac{(1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right]}{2 \times 2^{3/4} \sqrt{1+2 x^2+2 x^4}} + \\
 & \left((1-3 \sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \\
 & \left(2 \times 2^{3/4} (2-3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4} \right) + \\
 & \left(3 (3+\sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2 x^2+2 x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticPi}\left[\frac{1}{24} (12-11 \sqrt{2}), \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \left(8 \times 2^{3/4} (2-3 \sqrt{2}) \sqrt{1+2 x^2+2 x^4} \right)
 \end{aligned}$$

Result (type 4, 127 leaves):

$$\begin{aligned}
 & - \left(\left(\sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \right. \right. \\
 & \quad \left((1+i) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] - (1+4 i) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] + \right. \\
 & \quad \left. \left. 3 i \operatorname{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] \right) \right) / \left(4 \sqrt{1-i} \sqrt{1+2 x^2+2 x^4} \right)
 \end{aligned}$$

Problem 338: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}} dx$$

Optimal (type 4, 247 leaves, 3 steps):

$$-\frac{1}{4} \sqrt{\frac{3}{5}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{5}{3}} x}{\sqrt{1+2x^2+2x^4}}\right] - \left((3+\sqrt{2}) (1+\sqrt{2} x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \left(14 \times 2^{3/4} \sqrt{1+2x^2+2x^4} \right) + \left((3+\sqrt{2})^2 (1+\sqrt{2} x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2} x^2)^2}} \operatorname{EllipticPi}\left[\frac{1}{24} (12-11\sqrt{2}), 2 \operatorname{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} (2-\sqrt{2})\right] \right) / \left(56 \times 2^{1/4} \sqrt{1+2x^2+2x^4} \right)$$

Result (type 4, 99 leaves):

$$\frac{1}{4 \sqrt{1+2x^2+2x^4}} (1-i)^{3/2} \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \left(\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] - \operatorname{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i} x\right], i\right] \right)$$

Problem 339: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}} dx$$

Optimal (type 4, 245 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}x}}{\sqrt{1+2x^2+2x^4}}\right]}{2\sqrt{15}} + \left((3+\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticF}\left[2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(14 \times 2^{1/4} \sqrt{1+2x^2+2x^4} \right) - \left((3+\sqrt{2})^2 (1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \right) \text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), 2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] / \left(84 \times 2^{1/4} \sqrt{1+2x^2+2x^4} \right)$$

Result (type 4, 80 leaves):

$$- \left(\left(i \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \text{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i \text{ArcSinh}[\sqrt{1-i}x], i\right] \right) / \left(3 \sqrt{1-i} \sqrt{1+2x^2+2x^4} \right) \right)$$

Problem 340: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}} dx$$

Optimal (type 4, 399 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3(1+\sqrt{2}x^2)} - \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right]}{3\sqrt{15}} - \frac{1}{3\sqrt{1+2x^2+2x^4}} \\
 & 2^{1/4}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] + \\
 & \left((5-3\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticF}\left[2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
 & \left(21 \times 2^{3/4} \sqrt{1+2x^2+2x^4} \right) + \left((3+\sqrt{2})^2 (1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \right. \\
 & \left. \text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), 2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(126 \times 2^{1/4} \sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Result (type 4, 147 leaves):

$$\begin{aligned}
 & -\left(\left(i \left(-3i(1+2x^2+2x^4) + \sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2} \right. \right. \right. \\
 & \left. \left. \left(3\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - 3\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \right. \right. \right. \\
 & \left. \left. \left. (1+i)\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] \right) \right) \right) / \left(9x\sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Problem 341: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}} dx$$

Optimal (type 4, 422 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{2\sqrt{1+2x^2+2x^4}}{3x} - \frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3(1+\sqrt{2}x^2)} + \frac{2\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right]}{9\sqrt{15}} + \\
 & \frac{1}{3\sqrt{1+2x^2+2x^4}} 2 \times 2^{1/4} (1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] - \\
 & \left((1+19\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
 & \left(63 \times 2^{1/4} \sqrt{1+2x^2+2x^4} \right) - \left((3+\sqrt{2})^2 (1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \right. \\
 & \left. \text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), 2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(189 \times 2^{1/4} \sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Result (type 4, 219 leaves):

$$\begin{aligned}
 & \frac{1}{27x^3\sqrt{1+2x^2+2x^4}} \left(-3+12x^2+30x^4+36x^6+ \right. \\
 & 18i\sqrt{1-i}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \\
 & (3+15i)\sqrt{1-i}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] + \\
 & \left. 2(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] \right)
 \end{aligned}$$

Problem 348: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx$$

Optimal (type 4, 449 leaves, 10 steps):

$$\begin{aligned}
 & \frac{x^3 (1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20} x \sqrt{1+2x^2+2x^4} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}(1+\sqrt{2}x^2)} + \frac{27}{80} \sqrt{\frac{3}{5}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \\
 & \frac{(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]}{10 \times 2^{3/4} \sqrt{1+2x^2+2x^4}} + \\
 & \left((-2+7\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
 & \left(8 \times 2^{3/4} (-2+3\sqrt{2}) \sqrt{1+2x^2+2x^4} \right) + \\
 & \left(27(3+\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \operatorname{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), \right. \right. \\
 & \left. \left. 2 \operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(80 \times 2^{3/4} (2-3\sqrt{2}) \sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Result (type 4, 199 leaves):

$$\begin{aligned}
 & \left(4x + 12x^3 - 4i\sqrt{1-i}\sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \right. \\
 & (29-33i)\sqrt{1-i}\sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] + \\
 & 27(1-i)^{3/2} \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \\
 & \left. \operatorname{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i \operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] \right) / \left(80 \sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Problem 349: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx$$

Optimal (type 4, 423 leaves, 8 steps):

$$\frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}(1+\sqrt{2}x^2)} - \frac{9}{40}\sqrt{\frac{3}{5}}\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticE}\left[2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]}{10\times 2^{3/4}\sqrt{1+2x^2+2x^4}} - \left(\frac{(2^{1/4}+2^{3/4})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right)}{\left(8(-2+3\sqrt{2})\sqrt{1+2x^2+2x^4}\right) - \left(9(3+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), 2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]\right)}\left/\left(40\times 2^{3/4}(2-3\sqrt{2})\sqrt{1+2x^2+2x^4}\right)\right)$$

Result (type 4, 199 leaves):

$$\left(2x-4x^3-2i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] + (8-6i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - 9(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right]\right)\left/\left(40\sqrt{1+2x^2+2x^4}\right)\right)$$

Problem 350: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx$$

Optimal (type 4, 422 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}(1+\sqrt{2}x^2)} + \frac{3}{20}\sqrt{\frac{3}{5}}\operatorname{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] - \\
 & \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right]}{10\times 2^{3/4}\sqrt{1+2x^2+2x^4}} + \\
 & \left((2+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
 & \left(4\times 2^{3/4}(-2+3\sqrt{2})\sqrt{1+2x^2+2x^4} \right) + \\
 & \left(3(3+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), \right. \right. \\
 & \left. \left. 2\operatorname{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(20\times 2^{3/4}(2-3\sqrt{2})\sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Result (type 4, 199 leaves):

$$\begin{aligned}
 & -\left(\left(4x+2x^3+i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] + \right. \right. \\
 & \quad (1-2i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \\
 & \quad \left. 3(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\operatorname{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\operatorname{ArcSinh}\left[\sqrt{1-i}x\right], i\right] \right) / \left(20\sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Problem 351: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx$$

Optimal (type 4, 423 leaves, 8 steps):

$$\frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{5(1+\sqrt{2}x^2)} - \frac{1}{10}\sqrt{\frac{3}{5}}\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right] +$$

$$\frac{1}{5\sqrt{1+2x^2+2x^4}}2^{1/4}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticE}\left[2\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}(2-\sqrt{2})\right] -$$

$$\left(\left(2^{1/4}+2^{3/4}\right)(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}(2-\sqrt{2})\right]\right)/$$

$$\left(4(-2+3\sqrt{2})\sqrt{1+2x^2+2x^4}\right) -$$

$$\left(\left(3+\sqrt{2}\right)(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}),\right.\right.$$

$$\left.\left.2\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}(2-\sqrt{2})\right]\right)/\left(10\times 2^{3/4}(2-3\sqrt{2})\sqrt{1+2x^2+2x^4}\right)$$

Result (type 4, 199 leaves):

$$\left(6x+8x^3+4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right],i\right]-\right.$$

$$\left.(1+3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right],i\right]-\right.$$

$$\left.2(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\right.$$

$$\left.\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\text{ArcSinh}\left[\sqrt{1-i}x\right],i\right]\right)/\left(20\sqrt{1+2x^2+2x^4}\right)$$

Problem 352: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx$$

Optimal (type 4, 422 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}(1+\sqrt{2}x^2)} + \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right]}{5\sqrt{15}} - \\
 & \frac{3(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right]}{5 \times 2^{3/4} \sqrt{1+2x^2+2x^4}} + \\
 & \left((2+\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticF}\left[2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
 & \left(2 \times 2^{3/4} (-2+3\sqrt{2}) \sqrt{1+2x^2+2x^4} \right) + \\
 & \left((3+\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), \right. \right. \\
 & \left. \left. 2\text{ArcTan}[2^{1/4}x], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(15 \times 2^{3/4} (2-3\sqrt{2}) \sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Result (type 4, 199 leaves):

$$\begin{aligned}
 & \left(-6x - 18x^3 - 9i\sqrt{1-i} \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \text{EllipticE}\left[i\text{ArcSinh}[\sqrt{1-i}x], i\right] + \right. \\
 & (6+3i)\sqrt{1-i} \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \text{EllipticF}\left[i\text{ArcSinh}[\sqrt{1-i}x], i\right] + \\
 & \left. 2(1-i)^{3/2} \sqrt{1+(1-i)x^2} \sqrt{1+(1+i)x^2} \right. \\
 & \left. \text{EllipticPi}\left[\frac{1}{3} + \frac{i}{3}, i\text{ArcSinh}[\sqrt{1-i}x], i\right] \right) / \left(30\sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Problem 353: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx$$

Optimal (type 4, 468 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{x}{3\sqrt{1+2x^2+2x^4}} + \frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}} - \\
 & \frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{15(1+\sqrt{2}x^2)} - \frac{2\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right]}{15\sqrt{15}} - \\
 & \left(2 \times 2^{1/4} (1 + \sqrt{2} x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
 & \left(15\sqrt{1+2x^2+2x^4} \right) + \\
 & \left((-7+3\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \\
 & \left(3 \times 2^{3/4} (-2+3\sqrt{2}) \sqrt{1+2x^2+2x^4} \right) - \\
 & \left(2^{1/4} (3+\sqrt{2})(1+\sqrt{2}x^2) \sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}} \text{EllipticPi}\left[\frac{1}{24}(12-11\sqrt{2}), \right. \right. \\
 & \left. \left. 2\text{ArcTan}\left[2^{1/4}x\right], \frac{1}{4}(2-\sqrt{2})\right] \right) / \left(45(2-3\sqrt{2})\sqrt{1+2x^2+2x^4} \right)
 \end{aligned}$$

Result (type 4, 211 leaves):

$$\begin{aligned}
 & \frac{1}{90x\sqrt{1+2x^2+2x^4}} \\
 & \left(-12i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \right. \\
 & \left. (27-39i)\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right] - \right. \\
 & \left. 2\left(15+39x^2+12x^4 + \right. \right. \\
 & \left. \left. 2(1-i)^{3/2}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i\text{ArcSinh}\left[\sqrt{1-i}x\right], i\right]\right) \right)
 \end{aligned}$$

Problem 361: Unable to integrate problem.

$$\int \frac{x^4 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 390 leaves, 10 steps):

$$\frac{x \sqrt{d+e x^2}}{2 c} - \left(\left(b c d - b^2 e + a c e - \frac{b^2 c d - 2 a c^2 d - b^3 e + 3 a b c e}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right] \right) /$$

$$\left(c^2 \sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} \right) -$$

$$\left(\left(b c d - b^2 e + a c e + \frac{b^2 c d - 2 a c^2 d - b^3 e + 3 a b c e}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right] \right) /$$

$$\left(c^2 \sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} \right) + \frac{(c d - 2 b e) \text{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{2 c^2 \sqrt{e}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4 \sqrt{d+e x^2}}{a+b x^2+c x^4} dx$$

Problem 362: Unable to integrate problem.

$$\int \frac{x^2 \sqrt{d+e x^2}}{a+b x^2+c x^4} dx$$

Optimal (type 3, 324 leaves, 9 steps):

$$\frac{\left(c d - b e - \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right]}{c \sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e}} +$$

$$\frac{\left(c d - b e + \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right]}{c \sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e}} + \frac{\sqrt{e} \text{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{c}$$

Result (type 8, 31 leaves):

$$\int \frac{x^2 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Problem 363: Unable to integrate problem.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 240 leaves, 11 steps):

$$\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e} x}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e} x}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Problem 364: Unable to integrate problem.

$$\int \frac{\sqrt{d+ex^2}}{x^2 (a+bx^2+cx^4)} dx$$

Optimal (type 3, 291 leaves, 8 steps):

$$\frac{\sqrt{d+ex^2}}{ax} - \frac{c \left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e} x}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}}\right]}{a \sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}} - \frac{c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e} x}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}}\right]}{a \sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}}$$

Result (type 8, 31 leaves):

$$\int \frac{\sqrt{d+e x^2}}{x^2 (a+b x^2+c x^4)} dx$$

Problem 365: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}}{x^4 (a+b x^2+c x^4)} dx$$

Optimal (type 3, 373 leaves, 12 steps):

$$\begin{aligned} & -\frac{\sqrt{d+e x^2}}{3 a x^3} + \frac{2 e \sqrt{d+e x^2}}{3 a d x} + \frac{(b d-a e) \sqrt{d+e x^2}}{a^2 d x} + \\ & \frac{c \left(b d-a e + \frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}} \right) \operatorname{ArcTan} \left[\frac{\sqrt{2 c d-(b-\sqrt{b^2-4 a c}) e x}}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{a^2 \sqrt{b-\sqrt{b^2-4 a c}} \sqrt{2 c d-(b-\sqrt{b^2-4 a c}) e}} + \\ & \frac{c \left(b d-a e - \frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}} \right) \operatorname{ArcTan} \left[\frac{\sqrt{2 c d-(b+\sqrt{b^2-4 a c}) e x}}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{a^2 \sqrt{b+\sqrt{b^2-4 a c}} \sqrt{2 c d-(b+\sqrt{b^2-4 a c}) e}} \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{\sqrt{d+e x^2}}{x^4 (a+b x^2+c x^4)} dx$$

Problem 366: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}}{x^6 (a+b x^2+c x^4)} dx$$

Optimal (type 3, 512 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \\
 & \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} - \frac{(b^2d-acd-abe)\sqrt{d+ex^2}}{a^3dx} - \\
 & \left(c \left(b^2d-acd-abe + \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}} \right] \right) / \\
 & \left(a^3 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-(b-\sqrt{b^2-4ac})e} \right) - \\
 & \left(c \left(b^2d-acd-abe - \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}} \right] \right) / \\
 & \left(a^3 \sqrt{b+\sqrt{b^2-4ac}} \sqrt{2cd-(b+\sqrt{b^2-4ac})e} \right)
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{\sqrt{d+ex^2}}{x^6 (a+bx^2+cx^4)} dx$$

Problem 371: Unable to integrate problem.

$$\int \frac{x^4 (d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 595 leaves, 17 steps):

$$\begin{aligned}
 & \frac{(3 c d-4 b e) x \sqrt{d+e x^2}}{8 c^2} + \frac{x (d+e x^2)^{3/2}}{4 c} - \\
 & \left(\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} \left(b c d - b^2 e + a c e - \frac{b^2 c d - 2 a c^2 d - b^3 e + 3 a b c e}{\sqrt{b^2 - 4 a c}} \right) \right. \\
 & \quad \left. \operatorname{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right] \right) / \left(2 c^3 \sqrt{b - \sqrt{b^2 - 4 a c}} \right) - \\
 & \left(\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} \left(b c d - b^2 e + a c e + \frac{b^2 c d - 2 a c^2 d - b^3 e + 3 a b c e}{\sqrt{b^2 - 4 a c}} \right) \right. \\
 & \quad \left. \operatorname{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right] \right) / \\
 & \left(2 c^3 \sqrt{b + \sqrt{b^2 - 4 a c}} \right) + \frac{d (3 c d - 4 b e) \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{8 c^2 \sqrt{e}} - \\
 & \frac{\sqrt{e} \left(b c d - b^2 e + a c e - \frac{b^2 c d - 2 a c^2 d - b^3 e + 3 a b c e}{\sqrt{b^2 - 4 a c}} \right) \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{2 c^3} - \\
 & \frac{\sqrt{e} \left(b c d - b^2 e + a c e + \frac{b^2 c d - 2 a c^2 d - b^3 e + 3 a b c e}{\sqrt{b^2 - 4 a c}} \right) \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{2 c^3}
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4 (d+e x^2)^{3/2}}{a+b x^2+c x^4} dx$$

Problem 372: Unable to integrate problem.

$$\int \frac{x^2 (d+e x^2)^{3/2}}{a+b x^2+c x^4} dx$$

Optimal (type 3, 491 leaves, 16 steps):

$$\frac{e x \sqrt{d+e x^2}}{2 c} + \left(\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} \left(c d - b e - \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}} \right) \right. \\ \left. \operatorname{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right] \right) / \left(2 c^2 \sqrt{b - \sqrt{b^2 - 4 a c}} \right) + \\ \left(\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} \left(c d - b e + \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}} \right) \right. \\ \left. \operatorname{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right] \right) / \left(2 c^2 \sqrt{b + \sqrt{b^2 - 4 a c}} \right) + \\ \frac{d \sqrt{e} \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{2 c} + \frac{\sqrt{e} \left(c d - b e - \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}} \right) \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{2 c^2} + \\ \frac{\sqrt{e} \left(c d - b e + \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}} \right) \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{2 c^2}$$

Result (type 8, 31 leaves):

$$\int \frac{x^2 (d+e x^2)^{3/2}}{a+b x^2+c x^4} dx$$

Problem 373: Unable to integrate problem.

$$\int \frac{(d+e x^2)^{3/2}}{a+b x^2+c x^4} dx$$

Optimal (type 3, 487 leaves, 13 steps):

$$\left(\left(2c^2d^2 + b \left(b - \sqrt{b^2 - 4ac} \right) e^2 - 2ce \left(bd - \sqrt{b^2 - 4ac} d + ae \right) \right) \right. \\
 \left. \text{ArcTan} \left[\frac{\sqrt{2cd - \left(b - \sqrt{b^2 - 4ac} \right) ex}}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}} \right] \right) / \\
 \left(c \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{2cd - \left(b - \sqrt{b^2 - 4ac} \right) e} \right) - \\
 \left(\left(2c^2d^2 + b \left(b + \sqrt{b^2 - 4ac} \right) e^2 - 2ce \left(bd + \sqrt{b^2 - 4ac} d + ae \right) \right) \right) \\
 \left. \text{ArcTan} \left[\frac{\sqrt{2cd - \left(b + \sqrt{b^2 - 4ac} \right) ex}}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}} \right] \right) / \\
 \left(c \sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{2cd - \left(b + \sqrt{b^2 - 4ac} \right) e} \right) + \\
 \frac{\sqrt{e} \left(3cd - \left(b - \sqrt{b^2 - 4ac} \right) e \right) \text{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+ex^2}} \right]}{2c \sqrt{b^2 - 4ac}} - \\
 \frac{\sqrt{e} \left(3cd - \left(b + \sqrt{b^2 - 4ac} \right) e \right) \text{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+ex^2}} \right]}{2c \sqrt{b^2 - 4ac}}$$

Result (type 8, 28 leaves):

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Problem 374: Unable to integrate problem.

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)} dx$$

Optimal (type 3, 260 leaves, ? steps):

$$\begin{aligned}
 & - \frac{d \sqrt{d+ex^2}}{ax} - \frac{(2cd - (b - \sqrt{b^2 - 4ac})e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}x}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac} (b - \sqrt{b^2 - 4ac})^{3/2}} + \\
 & \frac{(2cd - (b + \sqrt{b^2 - 4ac})e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}x}{\sqrt{b + \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac} (b + \sqrt{b^2 - 4ac})^{3/2}}
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{(d+ex^2)^{3/2}}{x^2 (a+bx^2+cx^4)} dx$$

Problem 375: Unable to integrate problem.

$$\int \frac{(d+ex^2)^{3/2}}{x^4 (a+bx^2+cx^4)} dx$$

Optimal (type 3, 523 leaves, 19 steps):

$$\begin{aligned}
 & \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} + \left(\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e} \left(bd - ae + \frac{b^2d - 2acd - abe}{\sqrt{b^2 - 4ac}} \right) \right. \\
 & \left. \operatorname{ArcTan} \left[\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}x}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}} \right] \right) / \left(2a^2\sqrt{b - \sqrt{b^2 - 4ac}} \right) + \\
 & \left(\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e} \left(bd - ae - \frac{b^2d - 2acd - abe}{\sqrt{b^2 - 4ac}} \right) \right. \\
 & \left. \operatorname{ArcTan} \left[\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}x}{\sqrt{b + \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}} \right] \right) / \left(2a^2\sqrt{b + \sqrt{b^2 - 4ac}} \right) - \\
 & \frac{\sqrt{e}(bd-ae)\operatorname{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{a^2} + \frac{\sqrt{e}\left(bd-ae - \frac{b^2d-2acd-ab}{\sqrt{b^2-4ac}}\right)\operatorname{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{2a^2} + \\
 & \frac{\sqrt{e}\left(bd-ae + \frac{b^2d-2acd-ab}{\sqrt{b^2-4ac}}\right)\operatorname{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{2a^2}
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)} dx$$

Problem 381: Unable to integrate problem.

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 325 leaves, 9 steps):

$$\frac{x \sqrt{1-x^2}}{2c} + \frac{(2b+c) \operatorname{ArcSin}[x]}{2c^2} - \frac{\left(b^2 - ac + bc - \frac{b^3 - 3abc + b^2c - 2ac^2}{\sqrt{b^2 - 4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{c^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{b+2c-\sqrt{b^2-4ac}}} -$$

$$\frac{\left(b^2 - ac + bc + \frac{b^3 - 3abc + b^2c - 2ac^2}{\sqrt{b^2 - 4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{c^2 \sqrt{b+\sqrt{b^2-4ac}} \sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Problem 382: Unable to integrate problem.

$$\int \frac{x^2 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 263 leaves, 8 steps):

$$-\frac{\operatorname{ArcSin}[x]}{c} + \frac{\left(b+c - \frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{c \sqrt{b-\sqrt{b^2-4ac}} \sqrt{b+2c-\sqrt{b^2-4ac}}} +$$

$$\frac{\left(b+c + \frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{c \sqrt{b+\sqrt{b^2-4ac}} \sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^2 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Problem 383: Unable to integrate problem.

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 220 leaves, 9 steps):

$$\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} \operatorname{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}} \operatorname{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{\sqrt{b^2-4ac} \sqrt{b+\sqrt{b^2-4ac}}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Problem 384: Unable to integrate problem.

$$\int \frac{\sqrt{1-x^2}}{x^2 (a+bx^2+cx^4)} dx$$

Optimal (type 3, 265 leaves, 8 steps):

$$\frac{\sqrt{1-x^2}}{ax} - \frac{c \left(1 + \frac{2a+b}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{a \sqrt{b-\sqrt{b^2-4ac}} \sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c \left(1 - \frac{2a+b}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right]}{a \sqrt{b+\sqrt{b^2-4ac}} \sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Result (type 8, 31 leaves):

$$\int \frac{\sqrt{1-x^2}}{x^2 (a+bx^2+cx^4)} dx$$

Problem 385: Unable to integrate problem.

$$\int \frac{x^2 \sqrt{1-x^2}}{-1+x^2+x^4} dx$$

Optimal (type 3, 96 leaves, 8 steps):

$$-\text{ArcSin}[x] + \sqrt{\frac{1}{5} (2 + \sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2} (1 + \sqrt{5})} x}{\sqrt{1-x^2}}\right] - \sqrt{\frac{1}{5} (-2 + \sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2} (-1 + \sqrt{5})} x}{\sqrt{1-x^2}}\right]$$

Result (type 8, 27 leaves):

$$\int \frac{x^2 \sqrt{1-x^2}}{-1+x^2+x^4} dx$$

Problem 386: Unable to integrate problem.

$$\int \frac{x^8}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 479 leaves, 17 steps):

$$\begin{aligned} &-\frac{3 d x \sqrt{d+e x^2}}{8 c e^2} - \frac{b x \sqrt{d+e x^2}}{2 c^2 e} + \frac{x^3 \sqrt{d+e x^2}}{4 c e} - \\ &\frac{\left(b^3 - 2 a b c - \frac{b^4 - 4 a b^2 c + 2 a^2 c^2}{\sqrt{b^2 - 4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c})} e x}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}}\right]}{c^3 \sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b - \sqrt{b^2 - 4 a c})} e} - \\ &\frac{\left(b^3 - 2 a b c + \frac{b^4 - 4 a b^2 c + 2 a^2 c^2}{\sqrt{b^2 - 4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c})} e x}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}}\right]}{c^3 \sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b + \sqrt{b^2 - 4 a c})} e} + \\ &\frac{3 d^2 \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{8 c e^{5/2}} + \frac{b d \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{2 c^2 e^{3/2}} + \frac{(b^2 - a c) \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{c^3 \sqrt{e}} \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{x^8}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 387: Unable to integrate problem.

$$\int \frac{x^6}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 366 leaves, 13 steps):

$$\frac{x \sqrt{d+e x^2}}{2 c e} + \frac{\left(b^2 - a c - \frac{b(b^2-3 a c)}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b - \sqrt{b^2-4 a c}) e x}}{\sqrt{b - \sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{c^2 \sqrt{b - \sqrt{b^2-4 a c}} \sqrt{2 c d - (b - \sqrt{b^2-4 a c}) e}} +$$

$$\frac{\left(b^2 - a c + \frac{b(b^2-3 a c)}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e x}}{\sqrt{b + \sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{c^2 \sqrt{b + \sqrt{b^2-4 a c}} \sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e}} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{e x}}{\sqrt{d+e x^2}}\right]}{2 c e^{3/2}} - \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{e x}}{\sqrt{d+e x^2}}\right]}{c^2 \sqrt{e}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^6}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 388: Unable to integrate problem.

$$\int \frac{x^4}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 298 leaves, 10 steps):

$$-\frac{\left(b - \frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b - \sqrt{b^2-4 a c}) e x}}{\sqrt{b - \sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{c \sqrt{b - \sqrt{b^2-4 a c}} \sqrt{2 c d - (b - \sqrt{b^2-4 a c}) e}} -$$

$$\frac{\left(b + \frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e x}}{\sqrt{b + \sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{c \sqrt{b + \sqrt{b^2-4 a c}} \sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e x}}{\sqrt{d+e x^2}}\right]}{c \sqrt{e}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 389: Unable to integrate problem.

$$\int \frac{x^2}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 240 leaves, 6 steps):

$$\frac{\sqrt{b-\sqrt{b^2-4ac}} \operatorname{ArcTan}\left[\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right] + \sqrt{b+\sqrt{b^2-4ac}} \operatorname{ArcTan}\left[\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2-4ac} \sqrt{2cd-(b-\sqrt{b^2-4ac})e}} + \frac{\sqrt{b^2-4ac} \sqrt{2cd-(b+\sqrt{b^2-4ac})e}}{\sqrt{b^2-4ac} \sqrt{2cd-(b+\sqrt{b^2-4ac})e}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^2}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 390: Unable to integrate problem.

$$\int \frac{1}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 243 leaves, 5 steps):

$$\frac{2c \operatorname{ArcTan}\left[\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-(b-\sqrt{b^2-4ac})e}} - \frac{2c \operatorname{ArcTan}\left[\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2-4ac} \sqrt{b+\sqrt{b^2-4ac}} \sqrt{2cd-(b+\sqrt{b^2-4ac})e}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 391: Unable to integrate problem.

$$\int \frac{1}{x^2 \sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 280 leaves, 9 steps):

$$\frac{\frac{\sqrt{d+e x^2}}{a dx} + c \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \text{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})ex}}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{d+ex^2}}\right]}{a \sqrt{b - \sqrt{b^2-4ac}} \sqrt{2cd - (b - \sqrt{b^2-4ac})e}}$$

$$\frac{c \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \text{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2-4ac})ex}}{\sqrt{b + \sqrt{b^2-4ac}} \sqrt{d+ex^2}}\right]}{a \sqrt{b + \sqrt{b^2-4ac}} \sqrt{2cd - (b + \sqrt{b^2-4ac})e}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^2 \sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 392: Unable to integrate problem.

$$\int \frac{1}{x^4 \sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 341 leaves, 11 steps):

$$-\frac{\sqrt{d+e x^2}}{3 a d x^3} + \frac{b \sqrt{d+e x^2}}{a^2 d x} + \frac{2 e \sqrt{d+e x^2}}{3 a d^2 x} +$$

$$\frac{c \left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})ex}}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{d+ex^2}}\right]}{a^2 \sqrt{b - \sqrt{b^2-4ac}} \sqrt{2cd - (b - \sqrt{b^2-4ac})e}} + \frac{c \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2-4ac})ex}}{\sqrt{b + \sqrt{b^2-4ac}} \sqrt{d+ex^2}}\right]}{a^2 \sqrt{b + \sqrt{b^2-4ac}} \sqrt{2cd - (b + \sqrt{b^2-4ac})e}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^4 \sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 393: Unable to integrate problem.

$$\int \frac{1}{x^6 \sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 443 leaves, 14 steps):

$$-\frac{\sqrt{d+e x^2}}{5 a d x^5} + \frac{b \sqrt{d+e x^2}}{3 a^2 d x^3} + \frac{4 e \sqrt{d+e x^2}}{15 a d^2 x^3} - \frac{(b^2-a c) \sqrt{d+e x^2}}{a^3 d x} - \frac{2 b e \sqrt{d+e x^2}}{3 a^2 d^2 x}$$

$$\frac{8 e^2 \sqrt{d+e x^2}}{15 a d^3 x} - \frac{c \left(b^2 - a c + \frac{b(b^2-3 a c)}{\sqrt{b^2-4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2-4 a c}) e x}}{\sqrt{b - \sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{a^3 \sqrt{b - \sqrt{b^2-4 a c}} \sqrt{2 c d - (b - \sqrt{b^2-4 a c}) e}}$$

$$\frac{c \left(b^2 - a c - \frac{b(b^2-3 a c)}{\sqrt{b^2-4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e x}}{\sqrt{b + \sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{a^3 \sqrt{b + \sqrt{b^2-4 a c}} \sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^6 \sqrt{d+e x^2} (a+b x^2+c x^4)} dx$$

Problem 394: Unable to integrate problem.

$$\int \frac{x^6}{(d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 350 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{d^2 x}{e (c d^2 - b d e + a e^2) \sqrt{d + e x^2}} + \frac{2 \left(b^2 - a c - \frac{b (b^2 - 3 a c)}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}} \right]}{c \sqrt{b - \sqrt{b^2 - 4 a c}} (2 c d - (b - \sqrt{b^2 - 4 a c}) e)^{3/2}} + \\
 & \frac{2 \left(b^2 - a c + \frac{b (b^2 - 3 a c)}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}} \right]}{c \sqrt{b + \sqrt{b^2 - 4 a c}} (2 c d - (b + \sqrt{b^2 - 4 a c}) e)^{3/2}} + \frac{\text{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}} \right]}{c e^{3/2}}
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{x^6}{(d + e x^2)^{3/2} (a + b x^2 + c x^4)} dx$$

Problem 395: Unable to integrate problem.

$$\int \frac{x^4}{(d + e x^2)^{3/2} (a + b x^2 + c x^4)} dx$$

Optimal (type 3, 360 leaves, 8 steps):

$$\begin{aligned}
 & \frac{d x}{(c d^2 - b d e + a e^2) \sqrt{d + e x^2}} - \frac{\left(b d - a e - \frac{b^2 d - 2 a c d - a b e}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}} \right]}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} (c d^2 - b d e + a e^2)} \\
 & \frac{\left(b d - a e + \frac{b^2 d - 2 a c d - a b e}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}} \right]}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} (c d^2 - b d e + a e^2)}
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4}{(d + e x^2)^{3/2} (a + b x^2 + c x^4)} dx$$

Problem 396: Unable to integrate problem.

$$\int \frac{x^2}{(d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 333 leaves, 8 steps):

$$-\frac{e x}{(c d^2-b d e+a e^2) \sqrt{d+e x^2}} + \frac{c \left(d - \frac{b d-2 a e}{\sqrt{b^2-4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d-(b-\sqrt{b^2-4 a c}) e x}}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{2 c d-(b-\sqrt{b^2-4 a c}) e} (c d^2-b d e+a e^2)} +$$

$$\frac{c \left(d + \frac{b d-2 a e}{\sqrt{b^2-4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d-(b+\sqrt{b^2-4 a c}) e x}}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{2 c d-(b+\sqrt{b^2-4 a c}) e} (c d^2-b d e+a e^2)}$$

Result (type 8, 31 leaves):

$$\int \frac{x^2}{(d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Problem 397: Unable to integrate problem.

$$\int \frac{1}{(d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 341 leaves, 8 steps):

$$\frac{e^2 x}{d (c d^2-b d e+a e^2) \sqrt{d+e x^2}} - \frac{c \left(e - \frac{2 c d-b e}{\sqrt{b^2-4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d-(b-\sqrt{b^2-4 a c}) e x}}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{2 c d-(b-\sqrt{b^2-4 a c}) e} (c d^2-b d e+a e^2)} -$$

$$\frac{c \left(e + \frac{2 c d-b e}{\sqrt{b^2-4 a c}} \right) \text{ArcTan} \left[\frac{\sqrt{2 c d-(b+\sqrt{b^2-4 a c}) e x}}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{d+e x^2}} \right]}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{2 c d-(b+\sqrt{b^2-4 a c}) e} (c d^2-b d e+a e^2)}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{(d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Problem 398: Unable to integrate problem.

$$\int \frac{1}{x^2 (d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 339 leaves, 12 steps):

$$\frac{e (c d - b e) x}{a d (c d^2 + e (-b d + a e)) \sqrt{d+e x^2}} + \frac{-d - 2 e x^2}{a d^2 x \sqrt{d+e x^2}} -$$

$$\frac{2 c^2 \left(1 + \frac{b}{\sqrt{b^2 - 4 a c}}\right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right]}{a \sqrt{b - \sqrt{b^2 - 4 a c}} (2 c d - (b - \sqrt{b^2 - 4 a c}) e)^{3/2}} -$$

$$\frac{2 c^2 \left(1 - \frac{b}{\sqrt{b^2 - 4 a c}}\right) \text{ArcTan} \left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e x}}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d+e x^2}} \right]}{a \sqrt{b + \sqrt{b^2 - 4 a c}} (2 c d - (b + \sqrt{b^2 - 4 a c}) e)^{3/2}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^2 (d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Problem 399: Unable to integrate problem.

$$\int \frac{1}{x^4 (d+e x^2)^{3/2} (a+b x^2+c x^4)} dx$$

Optimal (type 3, 419 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{1}{3 a d x^3 \sqrt{d+e x^2}} + \frac{3 b d+4 a e}{3 a^2 d^2 x \sqrt{d+e x^2}} + \frac{2 e (3 b d+4 a e) x}{3 a^2 d^3 \sqrt{d+e x^2}} - \\
 & \frac{e (b c d-b^2 e+a c e) x}{a^2 d (c d^2+e (-b d+a e)) \sqrt{d+e x^2}} + \frac{2 c^2 \left(b+\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e x}}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{a^2 \sqrt{b-\sqrt{b^2-4 a c}}\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3 / 2}} + \\
 & \frac{2 c^2 \left(b-\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e x}}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{a^2 \sqrt{b+\sqrt{b^2-4 a c}}\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3 / 2}}
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^4 (d+e x^2)^{3 / 2} (a+b x^2+c x^4)} d x$$

Problem 400: Unable to integrate problem.

$$\int \frac{(f x)^m (d+e x^2)^q}{a+b x^2+c x^4} d x$$

Optimal (type 6, 243 leaves, 6 steps):

$$\begin{aligned}
 & \left(2 c (f x)^{1+m} (d+e x^2)^q \left(1+\frac{e x^2}{d}\right)^{-q} \operatorname{AppellF1}\left[\frac{1+m}{2}, 1, -q, \frac{3+m}{2}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{e x^2}{d}\right]\right) / \\
 & \left(\sqrt{b^2-4 a c}\left(b-\sqrt{b^2-4 a c}\right) f(1+m)\right) - \\
 & \left(2 c (f x)^{1+m} (d+e x^2)^q \left(1+\frac{e x^2}{d}\right)^{-q} \operatorname{AppellF1}\left[\frac{1+m}{2}, 1, -q, \frac{3+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, -\frac{e x^2}{d}\right]\right) / \\
 & \left(\sqrt{b^2-4 a c}\left(b+\sqrt{b^2-4 a c}\right) f(1+m)\right)
 \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{(f x)^m (d+e x^2)^q}{a+b x^2+c x^4} d x$$

Problem 405: Unable to integrate problem.

$$\int \frac{(d+e x^2)^q}{x (a+b x^2+c x^4)} d x$$

Optimal (type 5, 262 leaves, 8 steps):

$$\frac{\left(c \left(1 + \frac{b}{\sqrt{b^2 - 4 a c}} \right) (d + e x^2)^{1+q} \text{Hypergeometric2F1} \left[1, 1 + q, 2 + q, \frac{2 c (d + e x^2)}{2 c d - (b - \sqrt{b^2 - 4 a c}) e} \right] \right) / \left(2 a \left(2 c d - (b - \sqrt{b^2 - 4 a c}) e \right) (1 + q) \right) + \left(c \left(1 - \frac{b}{\sqrt{b^2 - 4 a c}} \right) (d + e x^2)^{1+q} \text{Hypergeometric2F1} \left[1, 1 + q, 2 + q, \frac{2 c (d + e x^2)}{2 c d - (b + \sqrt{b^2 - 4 a c}) e} \right] \right) / \left(2 a \left(2 c d - (b + \sqrt{b^2 - 4 a c}) e \right) (1 + q) \right) - \frac{(d + e x^2)^{1+q} \text{Hypergeometric2F1} \left[1, 1 + q, 2 + q, 1 + \frac{e x^2}{d} \right]}{2 a d (1 + q)}$$

Result (type 8, 29 leaves):

$$\int \frac{(d + e x^2)^q}{x (a + b x^2 + c x^4)} dx$$

Problem 407: Unable to integrate problem.

$$\int \frac{x^6 (d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Optimal (type 6, 339 leaves, 12 steps):

$$\frac{\left(\left(b^2 - a c - \frac{b (b^2 - 3 a c)}{\sqrt{b^2 - 4 a c}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \left(c^2 (b - \sqrt{b^2 - 4 a c}) \right) + \left(\left(b^2 - a c + \frac{b (b^2 - 3 a c)}{\sqrt{b^2 - 4 a c}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \left(c^2 (b + \sqrt{b^2 - 4 a c}) \right) - \frac{b x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{Hypergeometric2F1} \left[\frac{1}{2}, -q, \frac{3}{2}, -\frac{e x^2}{d} \right]}{c^2} + \frac{x^3 (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{Hypergeometric2F1} \left[\frac{3}{2}, -q, \frac{5}{2}, -\frac{e x^2}{d} \right]}{3 c}$$

Result (type 8, 29 leaves):

$$\int \frac{x^6 (d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Problem 408: Unable to integrate problem.

$$\int \frac{x^4 (d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Optimal (type 6, 273 leaves, 10 steps):

$$- \left(\left(\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{e x^2}{d} \right] \right) / \right. \\ \left. \left(c \left(b - \sqrt{b^2 - 4ac} \right) \right) \right) - \\ \left(\left(\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, -\frac{e x^2}{d} \right] \right) / \right. \\ \left. \left(c \left(b + \sqrt{b^2 - 4ac} \right) \right) \right) + \frac{x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{Hypergeometric2F1} \left[\frac{1}{2}, -q, \frac{3}{2}, -\frac{e x^2}{d} \right]}{c}$$

Result (type 8, 29 leaves):

$$\int \frac{x^4 (d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Problem 409: Unable to integrate problem.

$$\int \frac{x^2 (d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Optimal (type 6, 162 leaves, 6 steps):

$$\frac{x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{e x^2}{d} \right]}{\sqrt{b^2 - 4ac}} + \\ \frac{x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, -\frac{e x^2}{d} \right]}{\sqrt{b^2 - 4ac}}$$

Result (type 8, 29 leaves):

$$\int \frac{x^2 (d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Problem 410: Unable to integrate problem.

$$\int \frac{(d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Optimal (type 6, 190 leaves, 5 steps):

$$\begin{aligned}
 & - \left(\left(2 c x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \operatorname{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \right. \\
 & \quad \left. \left(b^2 - 4 a c - b \sqrt{b^2 - 4 a c} \right) \right) - \\
 & \left(2 c x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \operatorname{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \\
 & \quad \left(b^2 - 4 a c + b \sqrt{b^2 - 4 a c} \right)
 \end{aligned}$$

Result (type 8, 26 leaves):

$$\int \frac{(d + e x^2)^q}{a + b x^2 + c x^4} dx$$

Problem 411: Unable to integrate problem.

$$\int \frac{(d + e x^2)^q}{x^2 (a + b x^2 + c x^4)} dx$$

Optimal (type 6, 264 leaves, 10 steps):

$$\begin{aligned}
 & - \left(\left(c \left(1 + \frac{b}{\sqrt{b^2 - 4 a c}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \left(a \left(b - \sqrt{b^2 - 4 a c} \right) \right) \right) - \\
 & \left(c \left(1 - \frac{b}{\sqrt{b^2 - 4 a c}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \operatorname{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \\
 & \quad \left(a \left(b + \sqrt{b^2 - 4 a c} \right) \right) - \\
 & \frac{(d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -q, \frac{1}{2}, -\frac{e x^2}{d} \right]}{a x}
 \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(d + e x^2)^q}{x^2 (a + b x^2 + c x^4)} dx$$

Problem 412: Unable to integrate problem.

$$\int \frac{(d + e x^2)^q}{x^4 (a + b x^2 + c x^4)} dx$$

Optimal (type 6, 328 leaves, 12 steps):

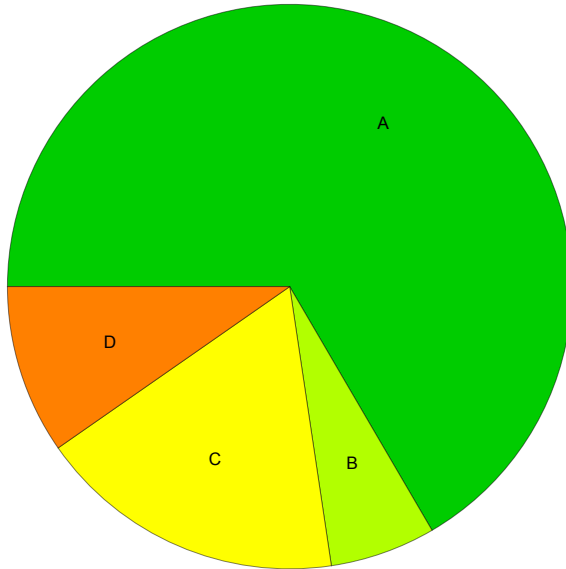
$$\begin{aligned} & \left(c \left(b + \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \\ & \left(a^2 \left(b - \sqrt{b^2 - 4 a c} \right) \right) + \\ & \left(c \left(b - \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}} \right) x (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{AppellF1} \left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, -\frac{e x^2}{d} \right] \right) / \\ & \left(a^2 \left(b + \sqrt{b^2 - 4 a c} \right) \right) - \frac{(d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{Hypergeometric2F1} \left[-\frac{3}{2}, -q, -\frac{1}{2}, -\frac{e x^2}{d} \right]}{3 a x^3} + \\ & \frac{b (d + e x^2)^q \left(1 + \frac{e x^2}{d} \right)^{-q} \text{Hypergeometric2F1} \left[-\frac{1}{2}, -q, \frac{1}{2}, -\frac{e x^2}{d} \right]}{a^2 x} \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(d + e x^2)^q}{x^4 (a + b x^2 + c x^4)} dx$$

Summary of Integration Test Results

413 integration problems



A - 275 optimal antiderivatives

B - 25 more than twice size of optimal antiderivatives

C - 73 unnecessarily complex antiderivatives

D - 40 unable to integrate problems

E - 0 integration timeouts